Anisotropic similarity, a constrained affine transformation: application to brain development analysis

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Synopsis

The study of brain development provides insights in the normal trend of brain evolution and enables early detection of abnormalities. We propose a method to quantify brain growth in three arbitrary orthogonal directions of the brain through linear registration. We introduce a 9 degrees of freedom transformation that gives the opportunity to extract scaling factors describing brain growth along those directions by registering a database of subjects in a common basis. We apply this framework to create a longitudinal curve of scaling ratios along fixed orthogonal directions from 0 to 16 years highlighting anisotropic brain development.

Introduction

In pediatric image analysis, the study of brain development provides insights in the normal trend of brain evolution and enables early detection of abnormalities. Tools like longitudinal atlases [1,2] allow to compute statistics on populations, understand brain variability at different ages to highlight changes in growth, shape, structure etc.
We propose a method to quantify brain growth in three arbitrary orthogonal directions of the brain through linear registration. We introduce a 9 degrees of freedom (dof) transformation called anisotropic similarity that is an affine transformation with constrained scaling directions along chosen vectors. This gives the opportunity to extract relative scaling factors describing brain growth along those directions by registering a database of subjects in a common basis. We apply this framework to create a longitudinal curve of scaling ratios along fixed orthogonal directions from 0 to 16 years highlighting anisotropic brain development.

Methods

In geometry, an affine transformation is a composition of a translation t and a linear map A operating on coordinates: $y = Ax + t$.

Using a particular singular value decomposition on the linear part A, we obtain:

$$A = RSU^T$$

Where U and R are matrices of the special orthogonal group and S is a diagonal matrix.

In a 3D space, an affine transformations has 12 dof. They can be decomposed as:

- a rotation (3 dof): U^T determines scaling directions.
- an anisotropic scaling (3 dof): S.
- a rotation (3 dof): matrix R
- a translation (3 dof): vector t.

We define an anisotropic similarity by fixing the U matrix column vectors such as they form an externally determined 3D orthonormal basis on which the scalings are applied.

In most linear registration algorithms based on local similarities, including block-matching [3,4] , the only step differing over the type of transformation is the estimation of the optimal global transformation from two sets of paired points. This often relies on finding the transformation minimizing the weighted sum of squared differences between those paired points. We developed a method to estimate the optimal anisotropic similarity.

Optimization:

Let $x = (x_1, \dots, x_N)$ and $y = (y_1, \dots, y_N)$ be two matrices containing (in column) the coordinates of N paired points.
Let $\bar{x} = \frac{1}{N}\sum_i^N x_i$ and $\bar{y} = \frac{1}{N}\sum_i^N y_i$.
Let $x'_i = x_i - \bar{x}$ and $y'_i = y_i - \bar{y} \qquad i = 1, \dots, N$.

The goal is to minimize the following least squares criterion:

$$C(R, S, t) = \sum_i \left\| y_i - (RSU^T x_i + t) \right\|^2$$

Where U is fixed.

Translation part:

For any transformation with linear part A and translational part t, the least squared problem associated to the matching of x and y consists in the minimization of the following criterion: $C(A, t) = \sum_i \|y_i - (Tx_i + t)\|^2$.
The optimal translation part can be obtained from the optimal linear part $\hat{A}$ and the barycenters of the two sets of points as:

$$\hat{t} = \bar{y} - \hat{A}\bar{x}$$

Figures

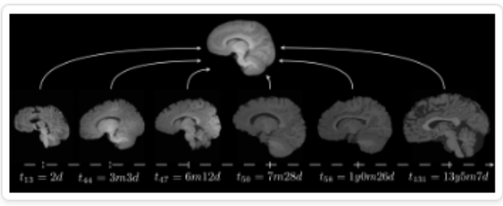

figure 1

All subjects are registered onto a single atlas created from them using an anisotropic similarity. The reference image and the imposed scaling directions are common for all those registrations. (y:years, m:months, d:days)

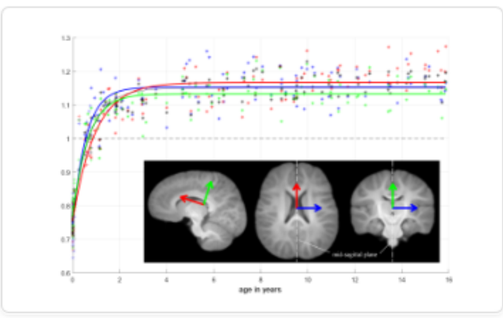

figure 2

Markers (∗ ASLpedia, + C-MIND and × Developing Human Connectome Project) represents scaling ratios of moving images over the reference image along the directions depicted on the sketch below it.

Curves fitting those data have been created using least squares regression on parameters of the following function: $f(x) = a + b\exp(cx + d)$. The black curve represents the overall growth model.

We can observe a growth at different speed depending on the direction followed by a stagnation.

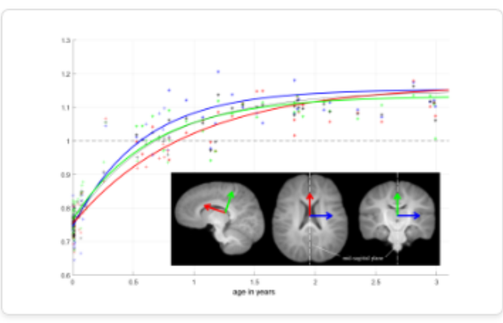

figure 3

This is close look on the 0 to 3 years range.

We can observe the the growth speed is not the same along the 3 directions.

The brain grow the slowest along the red direction, stagnation append around 5 years while it grow the fastest along the blue direction with a stagnation around 2.5 years.

This comfirms the observation that very young subjects seems to have a more istropic, spherical brain.

Linear part:

Let $\tilde{x} = U^T x$ and $\xi = S\tilde{x}$. R can be expressed using the associated quaternion q such as $R\xi_i = q * \xi_i * \bar{q}$ and matricial quaternions $Q_{y'_i}$ and P_{ξ_i} such as $y'_i * q = Q_{y'_i} q$ and $-q * \xi_i = -P_{\xi_i}^T q = P_{\xi_i} q$ [3] :

$$\begin{aligned}\tilde{C}(S, q) &= \sum_i ||y'_i - q * \xi_i * \bar{q}||^2 \\ &= -q^T \left(-\sum_i (Q_{y'_i} + P_{\xi_i})^2 \right) q\end{aligned}$$

For further computation, we denote $Z_i = (Q_{y'_i} + P_{\xi_i})^2$ and $Z = \sum_i Z_i$.
Separating the problem between S and q leads to an alternate optimization scheme, each step having an analytical solution:

- For a fixed value of S, estimate optimal rotation quaternion $\hat{q}$ as the eigen vector with the smallest eigenvalue of Z
- For a fixed value of q, estimate the optimal scaling matrix $\hat{S} = Diag(\hat{s}_1, \hat{s}_2, \hat{s}_3)$ following:

$$\frac{\partial \tilde{C}}{\partial s_j} = 0 \Leftrightarrow \hat{s}_j = \frac{1}{\sum_i \tilde{x}_{ji}^2} q^T \left(\sum_i Q_{y'_i} \frac{\partial P_{\xi_i}}{\partial s_j} \right) q$$

Results

144 subjects between 0 and 16 years old from:

- ASLpedia [a] : retrospective ASL study on routine pediatric MRI on 6 month to 16 years subjects (92)
- C-MIND [b] : subjects chosen in 0-1 years range (12)
- Developing Human Connectome Project [c] (40)

has been registered onto an atlas constituted of all those images using an anisotropic similarity (figure 1). We chose the direction matrix U defined earlier such as one column vector is orthogonal to the mid-sagittal plane [6] for symmetry reasons, the others corresponds to the principal directions of the voxel coordinates projected onto this plane. Then, we extracted the scaling ratios of the moving images over the reference one along those directions and used it to plot curves representing a growth model over time (figures 2 and 3).

Discussion

The choice of the direction matrix U is crucial. If our decision to pick a vector orthogonal to the mid-sagittal plane seems justified due to symmetry reasons, the selection of the other two are more debatable since they are based on purely geometric features ignoring iconic criterions.

Conclusion

We proposed a method to extract scaling factor alowing the quantification of anisotropic brain development. We have shown the relevance of this approach on the analysis of the growth model created form 155 pediatric subjects covering 0 to 16 years.

Acknowledgements

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a. Performed at Rennes Sud (France) hospital pediatric radiology department.

b. C-MIND Data Repository created by the C-MIND study of Normal Brain Development. This is a multisite, longitudinal study of typically developing children from ages newborn through young adulthood conducted by Cincinnati Children's Hospital Medical Center and UCLA and supported by the National Institute of Child Health and Human Development (Contract #s HHSN275200900018C). A listing of the participating sites and a complete listing of the study investigators can be found at:
https://cmind.research.cchmc.org/

c. The Developing Human Connectome Project (dHCP), is a highly collaborative, €15 million project led by King's College London, ImperialCollege London and Oxford University.
http://www.developingconnectome.org

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