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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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**Introduction**

**Objective**: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

**Approach**: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

**Material and Methods**

**Affine transformation**: $y = Lx + t$

$L = RSUT^T$

- $U$ is a rotation matrix that determines scaling directions
- $S$ is an anisotropic scaling (diagonal matrix)
- $R$ is a rotation matrix

**Anisotropic similarity**: affine transformation with fixed $U$

**Block-matching registration** [1][2]:

1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:

**Minimize**: $\bar{C}(R,S,t) = \sum_i ||y_i - (RSU^T x_i + t)||^2 , U$ fixed

$\bar{t} = \bar{y} - RSU^T \bar{x}$

$\bar{C}(R,S) = \sum_i ||y'_i - RS\bar{x}_i||^2 \quad y'_i = y_i - \bar{y}, \bar{x} = U^T(x_i - \bar{x})$

$\bar{C}(q,S) = \sum_i ||y'_i - q \times \xi_i||^2 \quad \xi_i = S\bar{x}_i$  [3]

**Matricial quaternions**: $y'_i = q \times q$ and $-q \times \xi_i = -P^{T}_{\xi}q = P_{\xi}q$

$\bar{C}(q,S) = -q^T \left( -\sum_i (Q_{q'_i} + P_{\xi})^2 q \right) q$

$A_i = (Q_{q'_i} + P_{\xi})^2$ and $A = \sum_i A_i$. Alternate optimization each having an analytical solution:

- For fixed $S$, $\bar{q}$ eigenvector with smallest eigenvalue of $A$
- For fixed $q$, $\bar{S}_{jj} = \frac{1}{\sum_i (Q_{q'_i} + P_{\xi})^2} q^T \left( \sum_i Q_{q'_i} \frac{\partial P_{\xi}}{\partial s_j} \right) q$

**Results**

**Scaling factors along chosen directions**

Black curve represents average model

Fitted using first order rational regression: $y = \frac{ax + b}{x + c}$

**Chosen scaling directions** fixed on the reference image:

1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen $U$.

Then we extracted the associated scaling factors describing brain growth.

**Conclusion**

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.

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