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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: $y = Lx + t$
$L = RSU^T$
- $U$ is a rotation matrix that determines scaling directions
- $S$ is an anisotropic scaling (diagonal matrix)
- $R$ is a rotation matrix

Anisotropic similarity: affine transformation with fixed $U$

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: $C(R,S,t) = \sum_i \|y_i - (RSU^T x_i + t)\|^2$, $U$ fixed

\[ \hat{t} = \bar{x} - R\bar{x}_S \]

\[ \hat{C}(R,S) = \sum_i \|y'_i - RS\bar{x}_i\|^2 \]
\[ y'_i = y_i - \bar{y}, \bar{x} = U^T(x_i - \bar{x}) \]

Matrical quaternions: $y'_i \ast q = Q y'_i q$ and $-q \ast \xi_i = -P^T \xi_i q = P \xi_i q$
$\hat{C}(q,S) = -q^T \left( -\sum_i (Q y'_i + P \xi_i)^2 \right) q$

\[ A_i = (Q y'_i + P \xi_i)^2 \text{ and } A = \sum_i A_i. \]

Alternate optimization each having an analytical solution:
- For fixed $S$, $\hat{q}$ eigenvector with smallest eigenvalue of $A$
- For fixed $q$, $\hat{S}_{ij} = \frac{1}{\sum_i \bar{x}_j^2} q^T \left( \sum_i Q y'_i \frac{\partial P \xi_i}{\partial s_j} \right) q$

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen $U$.

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: $y = \frac{ax + b}{x + c}$

Scaling factors over age in years

whole brain

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.