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To cite this version:

HAL Id: inserm-01871274
https://www.hal.inserm.fr/inserm-01871274
Submitted on 10 Sep 2018

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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: $y = Lx + t$
$L = R_SU^T$
- $U$ is a rotation matrix that determines scaling directions
- $S$ is an anisotropic scaling (diagonal matrix)
- $R$ is a rotation matrix

Anisotropic similarity: affine transformation with fixed $U$

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: $C(R,S,t) = \sum_i \|y_i - (RSU^T x_i + t)\|^2$, $U$ fixed

$\tilde{t} = g - RSU^T \tilde{x}$

$\hat{C}(R,S) = \sum_i \|y'_i - RS\tilde{x}_i\|^2$
$y'_i = y_i - \tilde{g}, \tilde{x} = U^T(x_i - \tilde{x})$

$\hat{C}(q,S) = \sum_i \|y'_i - q \cdot \xi_i \cdot \tilde{g}\|^2$
$\xi_i = S\tilde{x}_i$  [3]

Matricial quaternions: $y'_i \ast q = Q_{y_i}q$ and $-q \ast \xi_i = -P_{\xi_i}q = P_{\xi_i}q$

$\hat{C}(q,S) = -q^T \left( -\sum_i (Q_{y_i} + P_{\xi_i})^2 \right) q$

$A_i = (Q_{y_i} + P_{\xi_i})^2$ and $A = \sum_i A_i$.

Alternate optimization each having an analytical solution:
- For fixed $S$, $\tilde{q}$ eigenvector with smallest eigenvalue of $A$
- For fixed $q$, $\tilde{S}_{ij} = \sum_i \xi_i^T \frac{1}{A_{ij}} (\sum_i Q_{y_i} \frac{\partial P_{\xi_i}}{\partial s_j}) q$

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen $U$.

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: $y = \frac{ax + b}{x + c}$

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Scaling factors over age in years

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.
An interesting continuation is to compare the results among groups of population.