Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)

\( L = RSU^T \)

- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:

1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:

Minimize: \( C(R, S, t) = \sum_i \|y_i - (RSU^T x_i + t)\|^2 \) , \( U \) fixed

\( \tilde{t} = \tilde{g} - RSU^T \tilde{x} \)

\( \tilde{C}(R, S) = \sum_i \|y_i - RS\tilde{x}_i\|^2 \) \( y_i = y_i - \tilde{g}, \tilde{x} = U^T(x_i - \tilde{x}) \)

\( \tilde{C}(q, S) = \sum_i \|y_i' - q \cdot \xi_i \cdot \tilde{g}\|^2 \) \( \xi_i = S\tilde{x}_i \) [3]

Matrical quaternions: \( y_i' \cdot q = Q_{y_i} q \) and \( -q \cdot \xi_i = -P_{\xi_i} q = P_{\xi_i} q \)

\( \tilde{C}(q, S) = -q^T \left( -\sum_i (Q_{y_i} + P_{\xi_i})^2 \right) q \)

\( A_i = (Q_{y_i} + P_{\xi_i})^2 \) and \( A = \sum_i A_i \).

Alternate optimization each having an analytical solution:

- For fixed \( S \), \( \tilde{q} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \hat{S}_{jj} = \frac{1}{\sum_i \hat{q}^2_{ji}} q^{T} \left( \sum_i Q_{y_i} \frac{\partial P_{\xi_i}}{\partial s_j} \right) q \)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen \( U \).

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions

Black curve represents average model

Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:

1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.