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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)
\[ L = RSU^T \]
- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: \( C(R, S, t) = \sum_i \| y_i - (RSU^T x_i + t) \| ^2 \), \( U \) fixed

\[ \hat{t} = \bar{y} - RSU^T \bar{x} \]

\[ \tilde{C}(R, S) = \sum_i \| y_i' - RS\bar{x}_i \|^2 \]
\[ y_i' = y_i - \bar{y}, \quad \bar{x} = U^T(x_i - \bar{x}) \]

\[ \tilde{C}(q, S) = \sum_i \| y_i' = q \xi_i - q \bar{q} \| ^2 \]
\[ \xi_i = S\bar{x}_i \]

Matrical quaternions: \( y_i' = q \xi_i = - P_\xi q \) and \( q \xi_i = - P_\xi^T q = P_\xi q \)

\[ \tilde{C}(q, S) = - q^T \left( \sum_i (Q_{\xi_i} + P_\xi)^2 \right) q \]

\[ A_i = (Q_{\xi_i} + P_\xi)^2 \quad \text{and} \quad A = \sum_i A_i \]

Alternate optimization each having an analytical solution:
- For fixed \( S \), \( \hat{q} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \hat{S}_{jj} = \frac{1}{\sum_i \bar{x}_j^2} q^T \left( \sum_i Q_{\xi_i} \frac{\partial P_\xi}{\partial s_j} \right) q \)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen \( U \).

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.