Anisotropic similarity, a constrained affine transformation: Application to brain development analysis
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Antoine Legouhy¹, Olivier Commowick¹, François Rousseau² and Christian Barillot¹

¹ Univ Rennes, CNRS, INRIA, INSERM, IRISA UMR 6074, VISAGES ERL U-1228, F-35000, Rennes, France
² IMT Atlantique, LaTIM U1101 INSERM, UBL, Brest, France

Material and Methods

Affine transformation: \( y = Lx + t \)

\( L = RSU^T \)

- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:

Minimize: \( C(R, S, t) = \sum_i \| y_i - (RSU^T x_i + t) \|^2 \), \( U \) fixed

\( \hat{t} = \hat{y} - RSU^T \hat{x} \)

\( \hat{C}(R, S) = \sum_i \| y_i' - RS\hat{x}_i \|^2 \)

\( y_i' = y_i - \hat{y}, \hat{x} = U^T (x_i - \hat{x}) \)

\( \hat{C}(q, S) = \sum_i \| y_i' - q \cdot \xi_i \|^2 \)

\( \xi_i = S\hat{x}_i \) [3]

Matricial quaternions: \( y'_i \cdot q = Q_{xi} q \) and \( -q \cdot \xi_i = -P_{xi} q = P_{xi} q \)

\( \hat{C}(q, S) = -q^T \left( -\sum_i (Q_{xi} + P_{xi})^2 \right) q \)

\( A_i = (Q_{xi} + P_{xi})^2 \) and \( A = \sum_i A_i \).

Alternate optimization each having an analytical solution:
1. For fixed \( S \), \( \hat{q} \) eigenvector with smallest eigenvalue of \( A \)
2. For fixed \( q \), \( \hat{S}_{ji} = \frac{1}{\sum_i \hat{x}_j^2} \hat{q} q^T \left( \sum_i Q_{xi} \frac{\partial P_{xi}}{\partial s_j} \right) \)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen \( U \).

Then we extracted the associated scaling factors describing brain growth.

Scaling factors along chosen directions

Black curve represents average model

Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.