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To cite this version:

HAL Id: inserm-01871274
https://www.hal.inserm.fr/inserm-01871274

Submitted on 10 Sep 2018

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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)
\( L = RSU^T \)
- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: \( C(R, S, t) = \sum_i \| y_i - (RSU^T x_i + t) \|^2 \), \( U \) fixed
\( t = \frac{\bar{y} - R\bar{S} \bar{x}}{\bar{S}} \)
\( \tilde{C}(R, S) = \sum_i \| y_i' - R\tilde{S} \tilde{x}_i \|^2 \quad y_i' = y_i - \bar{y}, \tilde{x} = U^T(x_i - \bar{x}) \)
\( \tilde{C}(q, S) = \sum_i \| y_i' - q \xi_i - \bar{q} \|^2 \quad \xi_i = S\tilde{x}_i \quad [3] \)

Matricial quaternions: \( y_i' \ast q = Q_{y_i}'q \) and \(-q \ast \xi_i = -P_k^T q = P_k q \)
\( \tilde{C}(q, S) = -q^T \left( -\sum_i (Q_{y_i} + P_k)^2 \right) q \)
\( A_i = (Q_{y_i} + P_k)^2 \) and \( A = \sum_i A_i \).
Alternate optimization each having an analytical solution:
- For fixed \( S \), \( \tilde{q} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \tilde{S}_{ij} = \frac{1}{\sum_i \bar{x}_i^2} q^T \left( \sum_i Q_{y_i} \frac{\partial P_{k_i}}{\partial S_{ij}} \right) q \)

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.
An interesting continuation is to compare the results among groups of population.

References