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Measurements of ultrasound velocity and attenuation in numerical anisotropic porous media compared to Biot's and multiple scattering models

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Abstract

This article quantitatively investigates ultrasound propagation in numerical anisotropic porous media with finite-difference simulations in 3D. The propagation media consist of clusters of ellipsoidal scatterers randomly distributed in water, mimicking the anisotropic structure of cancellous bone. Velocities and attenuation coefficients of the ensemble-averaged transmitted wave (also known as the coherent wave) are measured in various configurations. As in real cancellous bone, one or two longitudinal modes emerge, depending on the micro-structure. The results are confronted with two standard theoretical approaches: Biot's theory, usually invoked in porous media, and the Independent Scattering Approximation (ISA), a classical first-order approach of multiple scattering theory. On the one hand, when only one longitudinal wave is observed, it is found that at porosities higher than 90% the ISA successfully predicts the attenuation coefficient (unlike Biot's theory), as well as the existence of negative dispersion. On the other hand, the ISA is not well suited to study two-wave propagation, unlike Biot's model, at least as far as wave speeds are concerned. No free fitting parameters were used for the application of Biot's theory. Finally we investigate the phase-shift between waves in the fluid and the solid structure, and compare them to Biot's predictions of in-phase and out-of-phase motions.

Keywords: Cancellous bone, Fast and slow waves, Porous media, Multiple scattering, FDTD simulations, Biot's theory

1. Introduction

Cancellous bone is a random and porous material with structural anisotropy. Ultrasound transmission experiments revealed that in some cases two compressional waves propagating at different velocities were observed, as reported both *in vitro* [1, 2, 3] and *in vivo* [4]. Yet this phenomenon is not fully understood.

From a theoretical point of view, there are several ways to account for wave propagation in media as complex as cancellous bone. A very simple approach, which will be referred to as Wood's model [5], consists in considering bone as a homogeneous medium in which sound speed is determined from the averaged mass densities and compliances of the solid and fluid phases, weighted by their respective volumetric fractions. A more elaborate theoretical description is given by Biot [6, 7]. Though it is out of the scope of this paper to give a full description of Biot's model, let us recall that it is an homogenization theory, like Wood's model. Biot's theory is a reference model for wave propagation in porous media, particularly because it was shown to predict successfully the velocities of two longitudinal waves in various porous media, from sintered glass spheres [8] to cortical [9] and cancellous bone [2, 10]. However, the validity of Biot's model is limited to low frequencies. Biot [7] himself wrote: "There remains however an upper bound for the frequency, namely, that at which the wavelength becomes of the order of the pore size. Such a case must, of course, be treated by a different method." Considering typical dimensions at stake, in the MHz domain, the ultrasonic wavelengths

are of the same order of magnitude as the size of the trabeculae [11, chap.1]. An immediate consequence is that single and even multiple scattering must be taken into account [12]. Scattering induce loss, which is not predicted by Biot's theory: as long as the fluid phase is free of absorption, Biot's fast and slow waves do not show any attenuation. Moreover, scattering does not only affect the wave amplitude, but also its velocity, though to a lesser degree. An alternative approach to Biot's would be to adopt a multiple scattering formalism for wave propagation in cancellous bone. A given sample is treated as one realization of a random process, whose typical physical parameters (density ρ , elastic moduli C_{ijkl}) randomly depend on position \vec{r} within the medium. Assuming gaussian statistics, the microstructure would be characterized by second-order moments i.e., correlation functions such as $\langle \rho(\vec{r})\rho(\vec{r}') \rangle$, $\langle C_{ijkl}(\vec{r})C_{mnop}(\vec{r}') \rangle$ etc. In the random multiple scattering approach, the wave field $s(\vec{r}, t)$ resulting from the emission of a pulse through a slice of bone is considered as a random variable. The basic quantities of interest are its statistical average $\langle s(\vec{r}, t) \rangle$ (also referred to in the literature as the "coherent wave") and variance. In particular, one important result of multiple scattering theory is that $\langle s(\vec{r}, t) \rangle$ follows Dyson's equation [13, 14]. If this equation can be solved, then the speed and attenuation can be inferred from a statistical description of the microstructure of any material. This was done for instance by Turner [15] in the framework of non-destructive characterization of polycrystals, where only one longitudinal mode was predicted and observed. The existence of two longitudinal modes within the framework of multiple scattering theory was reported by Cowan et al. [16]. They

58 have carried out experiments showing that two-wave propaga-113
 59 tion could occur in dense suspensions of plastic spherical scat-114
 60 terers, and that the slower of the two waves resulted from res-115
 61 onant coupling between scatterers. Their theoretical approach
 62 was the multiple scattering theory, under the Coherent Potential
 63 Approximation [17], assuming the wavelength was larger than
 64 the scatterers size.

65 Actually, none of the above-mentioned theories is perfectly
 66 suited to wave propagation in cancellous bone. Unlike poly-119
 67 crystals such as steel, cancellous bone has a solid and a fluid,
 68 phase, showing strong variations for both density and elas-121
 69 tic moduli. Moreover, the typical dimensions of the hetero-122
 70 geneities are not small compared to the wavelength, at least in
 71 the MHz range and above. One objective of this paper is to ex-124
 72 amine in what respect some typical results of Biot's, Wood's
 73 and multiple scattering theories can be useful to understand
 74 wave propagation in cancellous bone. For instance Wood's
 75 model is very simple and purely empirical but we will see that
 76 in some cases it may suffice to predict the velocity. As to Biot's
 77 theory, apart from its inadequacy to describe scattering, it re-130
 78 quires many parameters that are difficult to measure in the case
 79 of cancellous bone, especially *in vivo*. As a consequence, multi-132
 80 parameter fitting of experimental results is required, which im-133
 81 pairs the reliability of Biot's theory in the context of cancellous
 82 bone. To our knowledge Biot's theory was first developed for
 83 isotropic media and does not clearly explain why the two waves
 84 are observed for an ultrasound propagation along the main ori-137
 85 entation of the cancellous bone and not when the propagation is
 86 orthogonal [18]. Yet again, even though it fails to describe the
 87 attenuation and anisotropy, the velocities predicted by Biot can
 88 be in fairly good agreement with experimental observations. As
 89 to multiple scattering theory, though it yields an exact equation
 90 for the coherent wave field, it is not ideal either because in or-143
 91 der to derive practical results, at some stage an approximation
 92 has to be made, which necessarily limits the range of validity.

93 In this paper, we will use the simplest of all multiple scattering
 94 models, known as the ISA (Independent Scattering Approxima-
 95 tion), and examine its applicability to predict scattering losses
 96 and dispersion in anisotropic porous structures. 147

97 For a better understanding of ultrasound propagation in can-148
 98 cellous bone, we have chosen to begin with a numerical study,
 99 which is particularly flexible. In the last ten years, numerical
 100 studies have been intensively used, both in real bone structures
 101 derived from X-ray computed tomography [19, 20, 21] or in nu-152
 102 merically synthesized media [22]. Our approach here is based
 103 on the synthesis of numerical random and biphasic structures,
 104 previously described in [23]: elliptic (2D) or ellipsoidal (3D)
 105 scatterers were randomly distributed in a fluid to form a slab of
 106 random medium. In this previous work, we had qualitatively
 107 studied the occurrence of the fast and slow waves depending on
 108 simple statistical medium parameters using a finite-difference
 109 time-domain (FDTD) tool¹ to simulate the propagation of elas-160
 110 tic waves. We observed similar results, at least qualitatively, in
 111 2D and in 3D. Four conditions were necessary to observe the
 112 two waves in our simple models: 163

- a certain range of solid fraction (around 30% to 70 % for our model)
- a propagation parallel to the main orientation (i.e. in the direction of the long axis of the scatterers)
- the elastic nature of the solid phase
- enough connectivity of the solid matrix along the direction of propagation

The first two points are consistent with previous observations in real cancellous bone [11, chap.11], justifying *a posteriori* the relevance of our simplified model as a first step to study ultrasound propagation in trabecular bone. The last two points, on the contrary, could only have been studied numerically, and were consistent with the hypothesis that the fast wave results from a guiding through the solid matrix, whereas the slow wave is traveling predominantly through the fluid.

In the present article, velocities and attenuation coefficients of the coherent waves are measured in 3D samples, whether one or two waves are observed. Measurements methods are detailed in Sec.2.2. FDTD simulation results are confronted to the Independent Scattering Approximation (see Sec.2.3) and Biot's theory. Comparisons are presented in Sec.3, when one (3.1) or two waves (3.2) are observed. Sec.3.3 presents a different approach based on wave propagation from a pointlike source in 2D media, in order to observe the progressive transition from one to two waves. Finally, Sec.4 investigates the nature of the two waves by different methods. First, limit cases are studied by observing the two waves when the fluid phase is replaced by vacuum or when the solid matrix is perfectly rigid. In these cases propagation occurs only in the solid (respectively fluid) phase, giving interesting clues on the propagation paths for the fast and slow waves. Finally, we compare the phase shifts between the simulated fast and slow waves to Biot's theory.

2. Material and Methods

2.1. Numerical simulations

In a previous paper [23] we introduced numerical models generated by a Monte Carlo method: ellipses (2D) or ellipsoids (3D) of solid aligned along the same direction were added progressively in water (overlap was allowed), in order to obtain anisotropic and biphasic maps with given solid fractions. Examples of 3D maps can be seen on figure 1. The half lengths of the principal axes of the scatterers (i.e. the ellipsoids) were $a = 350 \mu m$, $b = c = 50 \mu m$ (note that c does not exist in 2D). The material properties chosen for the solid part were those of typical compact bone, compressional velocity $c_L = 4 mm.\mu s^{-1}$, shear velocity $c_T = 1.8 mm.\mu s^{-1}$ and mass density $\rho_s = 1850 kg.m^{-3}$. The background medium had the properties of water, $V_{fluid} = 1.5 mm.\mu s^{-1}$ and mass density $\rho_f = 1000 kg.m^{-3}$. Ultrasound propagation was simulated by a FDTD software, SimSonic, developed by E. Bossy [24] and freely available for download on-line². Stresses and particle velocities can be obtained at each point of the medium. Perfectly

¹<http://www.simsonic.fr>

²<http://www.simsonic.fr>

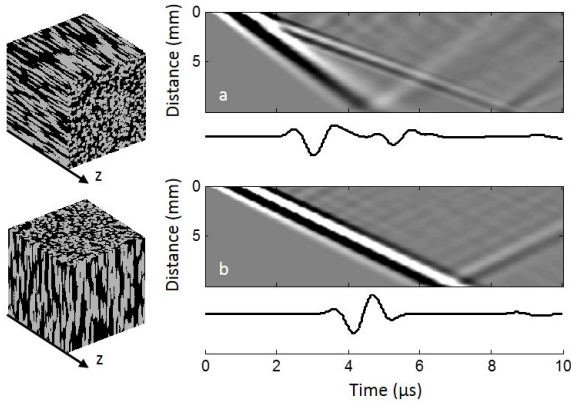


Figure 1: Left: Two typical 3D maps, both with a 50% solid fraction (gray). The propagation direction is along the z axis. (a) Time-distance diagram and corresponding time trace (taken at a 6 mm propagation distance) obtained when the ultrasound propagation is along the main direction. Two wavefronts are clearly distinguished. This simulation will be referred to as the "reference simulation". (b) Time-distance diagram and corresponding time trace (taken at a 6 mm propagation distance) obtained when the ultrasound propagation is perpendicular the main direction. Only one wave is observed.

matched layers (PML) were placed at the edges of the sample in the direction of propagation, while perfectly reflecting walls were placed in the transverse direction, so that the samples can be considered as semi-infinite slabs. The ultrasound pulses considered in the simulations had a central frequency of 1 MHz. The grid size was 10 micrometers in every directions, which corresponds to $1/150^{\text{th}}$ of the wavelength in the fluid. Taking advantage of the flexibility of numerical methods, we recorded the signals transmitted *inside* the samples for different depths within the propagation medium, in order to obtain full time-distance diagrams where the signal amplitude can be read as a function of time and propagation depth. This type of representation, not accessible experimentally where only time traces can be retrieved outside the sample, makes it easier to determine whether one or two waves are propagating. This is illustrated in Fig. 1: in the case of a propagation along the main direction, two waves are clearly observed (top) whereas only one wavefront appears in the case of a propagation perpendicular to the main direction, i.e. along the long axis of the ellipsoids (bottom). The transmitted signal is integrated over the whole transverse plane, for estimation of the *coherent wave*, assuming ergodicity. Due to the reflecting walls, the resulting signal is analogous to what would be measured in an ideal experiment with an infinite planar transducer placed within a semi-infinite scattering slab. More details on both the numerical samples and the simulations are available in [23]. In the rest of the present article, the simulation of propagation along the main direction in a binary medium made of bone and water with a 50% solid fraction (Fig. 1a) will be referred to as the "reference simulation".

2.2. Velocities and attenuation coefficients measurements

As the emitted signal is a pulse centered at 1 MHz with a -6 dB bandwidth of around 100%, we limited our study to the 0.5 MHz - 1.5 MHz frequency range. Theoretically the coherent

ent wave is obtained through ensemble averaging of the signals propagated through an infinite number of realizations, and is therefore only estimated here. As a consequence there is still an incoherent part remaining in the studied signal, mostly arriving after the ballistic wave(s). In order to remove it, the fast and slow waves were separated using a combination of rectangular and Hann windows, to avoid as much as possible the creation of disruptive frequencies brought by sharp cutting.

In a first step we focused on obtaining velocities and attenuation coefficients from the 3D numerical simulations. As detailed above, transmitted signals for different propagation distances were recorded and stored in a time-distance matrix $s(t, z)$. Performing Fast Fourier Transforms (FFT) of each column provides $\tilde{s}(\omega, z)$, a matrix where each row corresponds to $\tilde{s}_\omega(z)$ the signal as a function of propagation distance, at a given angular frequency ω . Phase velocity and attenuation coefficient were respectively obtained as follows:

- by taking the unwrapped phase angle $\varphi_\omega(z)$ of $\tilde{s}_\omega(z)$ we obtained:

$$\varphi_\omega(z) = \arg(\tilde{s}_\omega(z)) = \varphi_\omega(0) + kz \quad (1)$$

so $\varphi_\omega(z)$ is linear, with a slope equal to $k(\omega)$ giving easy access to the phase velocity $v(\omega)$:

$$v(\omega) = \frac{\omega}{k(\omega)} \quad (2)$$

- by taking the modulus of $\tilde{s}_\omega(z)$ and assuming an exponential decrease of the signal, one obtains the following expression involving the attenuation coefficient $\gamma(\omega)$:

$$|\tilde{s}_\omega(z)| = e^{-\gamma(\omega)z} \quad (3)$$

γ is then obtained by a linear fit of $\ln |\tilde{s}_\omega(z)|$ with z . Note that we chose to represent the attenuation coefficient for the amplitude γ as it is usually used in the field of bone quantitative ultrasound [11]. In the multiple scattering community, one usually refers to the scattering mean free path l_e , defined as the decay length for the intensity of the coherent field. The two parameters are simply related:

$$l_e = \frac{1}{2\gamma} \quad (4)$$

Frequency-resolved measurements of attenuation and velocities were obtained for the reference simulation (as exposed in Sec.3.2). However Eq.2 is inapplicable when the two wavefronts are not clearly separated. In such cases, fast and slow waves velocities were estimated through time-of-flight measurements, by tracking the first minimum of each wave in the time-distance diagrams, thus losing the frequency-dependence.

2.3. Independent Scattering Approximation (ISA)

In a random scattering medium the coherent field is the solution of Dyson's equation [13]. The key element in Dyson's equation is the so-called "self-energy" which wraps up all possible multiple scattering terms. The self-energy can be written

226 as a perturbative development of terms in $1/(k_0 l_e)$ [14]. The 249
 227 simplest approximation consists in keeping only the first-order 250
 228 term of the development. This is known as the "Independent 251
 229 Scattering Approximation" (ISA). 252

Here, the numerical samples can be seen as a random ar- 253
 rangement of identical scatterers in a lossless fluid with velocity 254
 c_0 (wavenumber $k_0 = \omega/c_0$). In that simple case, from a physi- 255
 cal point of view the ISA amounts to considering that the scat- 256
 terers positions are uncorrelated, and that the wave never loops 257
 back to a scatterer that has already been visited [25]. Under 258
 this assumption, the self-energy only depends on the scatterers 259
 concentration n and the scattering properties of one individual 260
 scatterer, particularly its angular directivity pattern $f(\theta)$. Then
 Dyson's equation has an analytical solution, and the coherent 261
 wavefield is characterized by a dispersion equation $k_{eff}(\omega)$,
 with k_{eff} the complex-valued "effective wave number". In other
 words, on average the effect of multiple scattering is to modify
 the speed as well as the attenuation of the medium, since k_{eff}
 is a complex number, unlike k_0 . Under the ISA, the effective
 wave number k_{eff} , and as a consequence velocity and attenua-
 tion, can be estimated from the density of scatterers n , the speed
 of sound in the surrounding fluid c_0 and the forward-scattered
 pressure $\tilde{\psi}_{scat}$, with

$$\tilde{\psi}_{scat}(\theta = 0, r) = \frac{e^{ik_0 r}}{r} f(\theta = 0) \quad (5)$$

This last parameter is obtained at a distance r in the direction
 of incidence ($\theta = 0$) when one single scatterer immersed in the
 fluid is insonified by a plane wave. The resulting dispersion
 relation is:

$$k_{eff}^2 = k_0^2 + 4\pi n f(\theta = 0) \quad (6)$$

where n is the solid fraction Φ_s divided by the volume of a 264
 single scatterer (a, b and c are the half axes defined in Sec. 2.1) 265

$$n = \frac{\Phi_s}{\frac{4}{3}\pi abc} \quad (7)$$

230 The forward scattered pressure $\tilde{\psi}_{scat}(\theta = 0, r)$ and $f(\theta = 0)$ 270
 231 are obtained numerically by a simple FDTD simulation where 271
 232 a plane wave is emitted in a medium containing only one scat- 272
 233 terer surrounded by water. The medium is 8 mm thick in the 273
 234 propagation direction and 16 mm \times 16 mm large in the trans- 274
 235 verse directions. The center of the scatterer is just ahead of the 275
 236 emitting boundary at a 500 μm depth, centered in each trans- 276
 237 verse direction. The signal is recorded at a 7.5 mm distance 277
 238 ahead of the center of the scatterer. We chose the same grid 278
 239 step size as for the random media simulations to account for the 279
 240 discretization effects at the border of the ellipsoid. 280

241 For the single scatterer simulation, boundary conditions were 281
 242 chosen strictly similar to those of the random media simulation 282
 243 (PML in the direction of propagation, symmetry conditions in 283
 244 transverse directions) to ensure a perfect incoming plane wave. 284
 245 The forward scattered signal had to be windowed to limit the 285
 246 contribution of image scatterers due to the symmetry condi- 286
 247 tions. As a reference, the same simulation was run with no 287
 248 scatterer. The resulting field $\psi_0(\theta = 0, r, t)$ was then subtracted 288

from the total field in order to obtain the forward scattered field
 $\psi_{scat}(\theta = 0, r, t)$. Finally, the Fourier transform of the scat-
 tered field was normalized by that of the incident wave to ob-
 tain $\tilde{\psi}_{scat}(\theta = 0, r)$ which comprises only the frequencies in the
 bandwidth of the incident pulse i.e., from 0.5 MHz to 1.5 MHz.

It should be noted that Eq.6 implies that there is only one
 effective number. Therefore intrinsically the ISA only predicts
 the existence of one longitudinal mode, with a velocity and an
 attenuation different from that of the fluid. However even when
 two waves are observed, it may be interesting to compare the
 velocity and attenuation predicted by the ISA with the simu-
 lated results.

2.4. Some predictions of Biot's theory

Biot's theory [6, 7] was originally developed for the study of
 ultrasound propagation in porous, isotropic rocks, with a low
 frequency assumption. Various groups have used Biot's frame-
 work in other fields of application and with additional hypothe-
 ses, providing in some cases the assessment of wave velocities
 from only a few parameters. For example, in the limit where the
 porous frame is much stiffer than the fluid, Johnson [26] gives
 simple relations (see Appendix A) from which we can derive
 the velocities of the fast and slow waves:

$$V_{fast} = \frac{V_{dry}^L}{\sqrt{1 + \frac{\Phi_f \rho_f}{\Phi_s \rho_s} \left(1 - \frac{1}{\alpha}\right)}} \quad (8)$$

$$V_{slow} = \frac{V_{fluid}}{\sqrt{\alpha}} \quad (9)$$

where Φ_f and Φ_s are the fluid and solid fractions, ρ_f and ρ_s
 the fluid and solid densities, V_{fluid} the speed of sound in the
 filling fluid. V_{dry}^L is the longitudinal speed of sound in the dry
 sample, i.e. the velocity when the fluid is replaced by vacuum,
 a situation which is easy to simulate numerically. Finally α is
 the geometric tortuosity, which is particularly difficult to assess
 in porous media and is by definition independent of material
 properties but depends on the micro-architecture.

Note that there is no frequency dependence in Eq.8 and 9,
 which is consistent with the use of time-of-flight measurements
 to estimate velocities from experimental results.

Biot's framework is also used in this study to gain insight on
 the origin of the fast and slow waves. One of the main conclu-
 sions of Biot's theory is that the fluid and the solid move
 either in phase (fast wave) or out of phase (slow wave) [6]. In
 order to check this prediction, the transverse plane is divided in
 two regions corresponding to solid and fluid zones respectively.
 The particle velocity is integrated separately in the two areas,
 as if the receiving transducer was only in contact with the fluid
 or with the solid. Then we can examine whether the resulting
 coherent waves in the fluid and in the solid exhibit a particular
 phase shift. This peculiar prediction cannot be verified exper-
 imentally, unless we could have a point-like transducer deep
 inside the sample measuring displacements in the fluid and in
 the solid. But numerical simulations make that measurement
 possible. Results are shown in Sec.4.2, for two different solid
 fractions (50 % and 70%).

289 3. From one to two waves

290 3.1. One wave

291 In a first step, we compare the frequency-dependence of at-
 292 tenuation coefficient and velocity (measured as described in
 293 Sec.2.2) to ISA predictions, in samples where only one com-
 294 pressional wave could be observed. As shown in [23] and Fig.1,
 295 this occurs when the ultrasound propagation is perpendicular to
 296 the main direction of the samples. The velocities and attenua-
 297 tion coefficients are plotted in Fig.2 for various solid fractions:
 298 10%, 30% and 50%.

As expected, the performance of ISA strongly depends on the
 density of scatterers. For a low solid fraction (10%) both veloci-
 ties and attenuation coefficients are well predicted by the ISA.
 The discrepancy increases with solid fraction. These results are
 in agreement with the fact that the ISA is a first-order approx-
 imation, which naturally fails as the solid fraction increases.
 Another interesting point is that a negative dispersion was ob-
 served for the three different bone fractions. In fact, the ob-
 served linear relationship between velocity and frequency, with
 a negative slope, was well predicted by the ISA. This negative
 dispersion is of particular interest because it has also been ob-
 served experimentally in cancellous bone [27]. The velocity
 increases, as expected, with the solid fraction. In the simple
 case of one wave propagation, Wood [5] theory could be used
 for estimating porosity based on velocity measurements. Ac-
 cording to Wood, the compressional wavespeed of an effective
 medium depends on 5 parameters: the fluid and solid densities
 ρ_f and ρ_s , the fluid and solid bulk moduli K_f and K_s and the
 fluid and solid fractions Φ_f and Φ_s :

$$V_{Wood} = \sqrt{\frac{K_f K_s}{(\Phi_f K_s + \Phi_s K_f)(\Phi_s \rho_s + \Phi_f \rho_f)}} \quad (10)$$

299 Fig.3 compares the velocities obtained in our samples (simply
 300 deduced from time-of-flight measurements in the time-distance
 301 diagrams) with the one predicted by Wood in a range of solid
 302 fraction going from 0 to 0.5.

303 There is a good agreement between the velocities measured
 304 with the simulations and predicted by Wood, especially for a
 305 low solid fraction. In fact as seen in Fig.2 the higher the solid
 306 fraction the higher the dispersion, so for higher solid fractions,
 307 the time-of-flight methods for velocity measurements probably
 308 become biased. However this result shows that in this configu-
 309 ration (propagation perpendicular to the main direction) micro-
 310 architecture parameters other than porosity do not seem to play
 311 a role in the velocities. In some cases where only one com-
 312 pressional wave was observed, Wood and Biot theories were
 313 found to yield similar values for the velocity [28]. As to the
 314 attenuation coefficient, it is worth noticing that the observed at-
 315 tenuation coefficient shows a power-law dependency with fre-
 316 quency, with characteristic exponents of 3.4, 2.5 and 2.1 for
 317 10%, 30% and 50% porosity. When increasing the solid frac-
 318 tion, the scatterers are more likely to overlap and create struc-
 319 tures significantly larger than the wavelength. The decay of
 320 the characteristic exponent is therefore consistent with predic-
 321 tions from scattering theories, where the attenuation coefficient

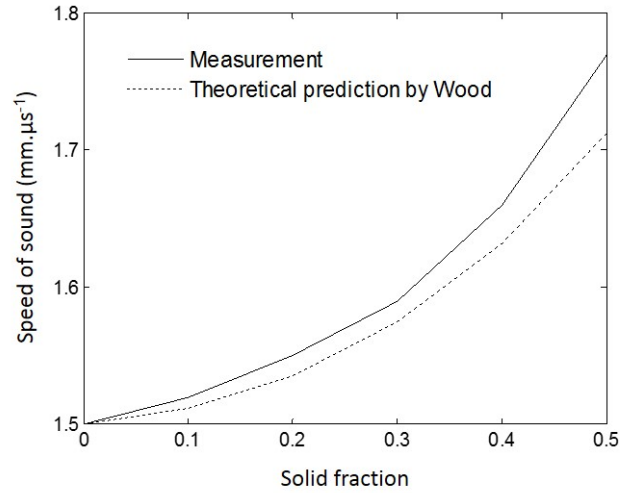


Figure 3: Speed of sound as a function of solid fraction (from 0 to 0.5) measured from time-of-flight measurements in the time-distance diagrams and predicted by Wood, using Eq.10

varies as ω^4 in the very low frequency regime and as ω^2 in the high-frequency regime. But this is very different from the usual linear dependence with frequency which has often been reported in cancellous bone [29, 30]. This indicates that our model samples exhibit some, but not all of the features typical of porous bone.

3.2. Two waves

We now move to the case where two compressional waves propagate. Previous work by our group showed that the two waves could be observed in the case of an ultrasound propagation along the main direction and for an appropriate range of solid fraction, from 30% to 70% [23] (Fig.1). The two-wave configuration is a little more difficult to study because it requires the two waves to be separated, in order to apply the routine described in Sec.2.2. We therefore limited the study to the case of a 50% solid fraction, where the two waves were found to be best separated. In addition, the propagation depth was increased to 20 mm in order to facilitate the distinction between the slow and the fast wave. The simulation results show that the two waves were conveniently separated for depths ranging between 6.7mm and 11.7mm, without being too strongly attenuated by scattering or polluted by reflected waves from the sample boundary. Hann windowing was used to isolate the slow wave. Fig.4 shows the resulting attenuation coefficients and velocities of the fast and slow waves, as well as the ISA predictions.

ISA only predicts one wave, as discussed in Sec.2.3. Furthermore the velocity predicted by ISA (around 1.5 mm.μs⁻¹) was far from the measured velocities for the fast (around 2.6 mm.μs⁻¹) and slow (around 1.3 mm.μs⁻¹) waves. This can be explained by the fact that the Independent Scattering Approximation sees the medium as a perturbation of the surrounding fluid (water with a speed of sound of 1.5 mm.μs⁻¹). Here,

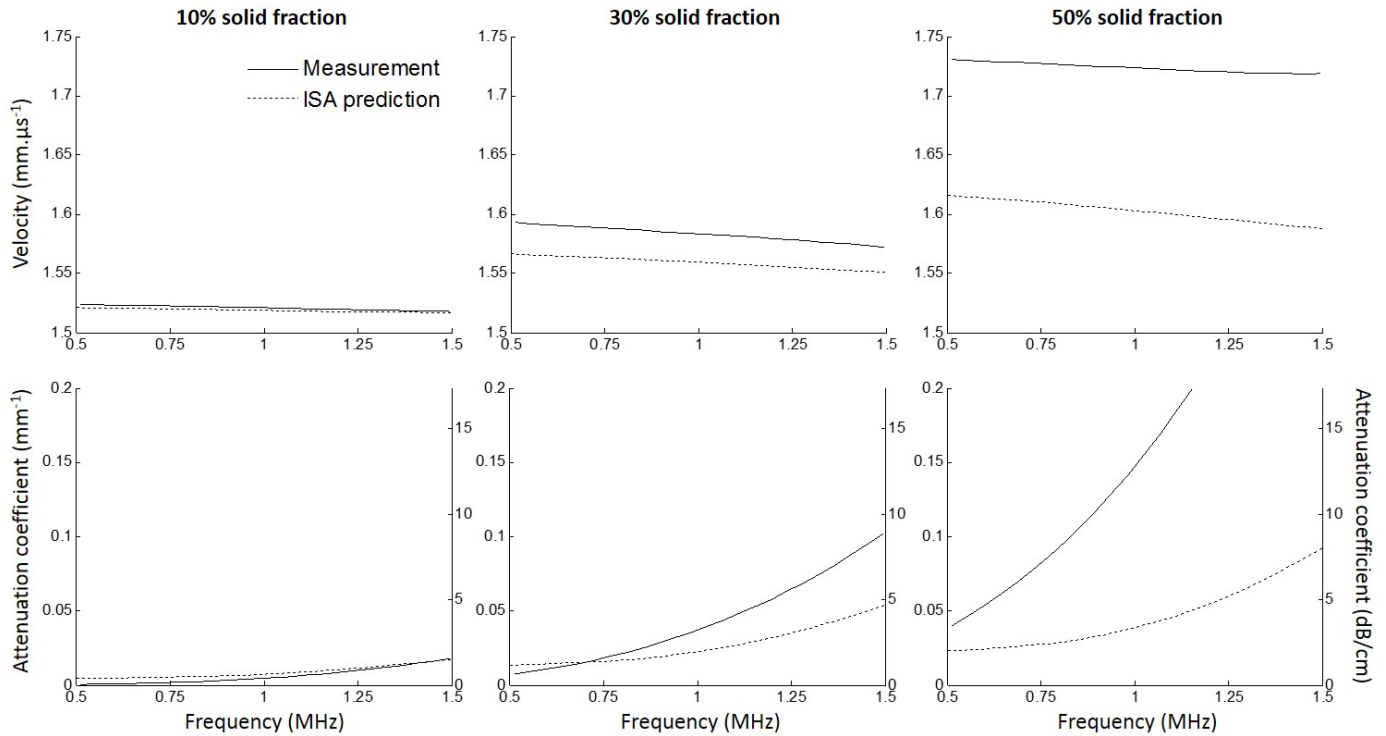


Figure 2: Velocities (top) and attenuation coefficients (bottom) measured in three different samples with a 10% (left), 30% (center) and 50% (right) solid fraction. For all three cases, ultrasound propagation was set perpendicular to the direction of anisotropy so that only one wave could be observed. The three velocity measurements exhibit a linear dependence of frequency with a negative dispersion (the slopes are -0.006 mm for the 10% solid fraction, -0.02 mm for the 30% one and -0.013 mm for the 50% one) quite well predicted by ISA (respectively -0.005 mm, -0.016 mm and -0.029 mm).

355 the bone fraction is far beyond the limit of use of this theory³ as₃₇₇
 356 already discussed in the previous subsection, but this is required₃₇₈
 357 to observe two separable waves for our numerical samples [23].₃₇₉
 358 Still, it is interesting to notice that fast and slow waves attenua-₃₈₀
 359 tion coefficients seem of the same order of magnitude. It should₃₈₁
 360 be pointed out that Biot's theory predicts no attenuation (loss-₃₈₂
 361 less fluid), whereas numerical results clearly show that both fast₃₈₃
 362 and slow waves undergo a strong attenuation due to scattering.₃₈₄
 363 The observed attenuation is significantly stronger than what the₃₈₅
 364 ISA predicts. It is also interesting to notice that both waves₃₈₆
 365 show a slight positive dispersion. This is consistent with previ-₃₈₇
 366 ous observations on cancellous bone, where no negative disper-₃₈₈
 367 sion was observed when the two waves were clearly separated₃₈₉
 368 [1][11, chap.5]. Finally, note that the slow wave velocity is₃₉₀
 369 slower than the speed of sound in water, which is in agreement₃₉₁
 370 with Eq.9, as tortuosity is real and greater than unity.

371 3.3. Source point

372 Let us now consider a different approach where a source₃₇₃
 373 point is placed at the center of the map (to avoid a possible nu-₃₇₄
 374 merical issue, we made sure this point lied in the fluid phase).₃₇₅
 375 Propagation can be studied along all directions simultaneously,₃₇₆
 376 giving much more information than the previous plane wave

simulations. In the point-source configuration, the coherent wave could not be estimated by spatial averaging anymore, and ensemble averaging over 50 realizations of the random medium was performed. As the computational cost is much higher, the simulations were carried out only in 2D. It has previously been shown that qualitative results were similar in 2D and in 3D [23]. Each map is 15 mm by 15 mm with a $5 - \mu\text{m}$ grid step. In order to account for the geometrical decay introduced by propagation from a source point, the signals were multiplied by \sqrt{r} , r being the distance from the point of observation to the source point. Fig.5 shows snapshots of the propagation after averaging over the 50 realizations. Clearly, the incoherent contribution has not yet been completely cancelled out.

Fig.5 shows a strong anisotropy of the propagation through the random samples. In the main direction (horizontal) two wavefronts can be distinguished, even though the ratio of coherent to incoherent wave amplitude would have benefited from averaging over a larger number of realizations. This result is in agreement with the previous observations. If we continuously rotate to the case where the propagation is perpendicular to the main axis, the fast and slow waves velocities are getting closer until only one wave can be distinguished. This observation rules out the possibility that the velocity of the slow wave continuously drops to zero when the propagation direction changes from parallel to perpendicular to the main axis. It also raises questions about the phenomenon actually taking place when only one wave can be observed: two waves could

³The ISA is valid as long as $k_0 \gg n\sigma$, with σ the total scattering cross-section of a single scatterer. The high-frequency limit for σ is twice the geometric cross-section

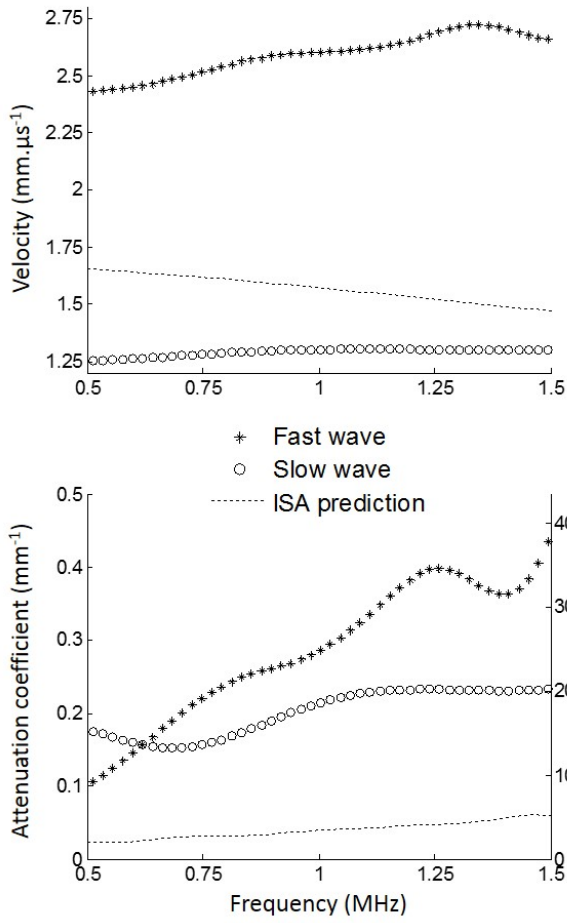


Figure 4: Velocities (top) and attenuation coefficients (bottom) of the fast and slow waves, measured in a 50% solid fraction sample in the case of ultrasound propagation along the main direction. Confrontation with the ISA prediction

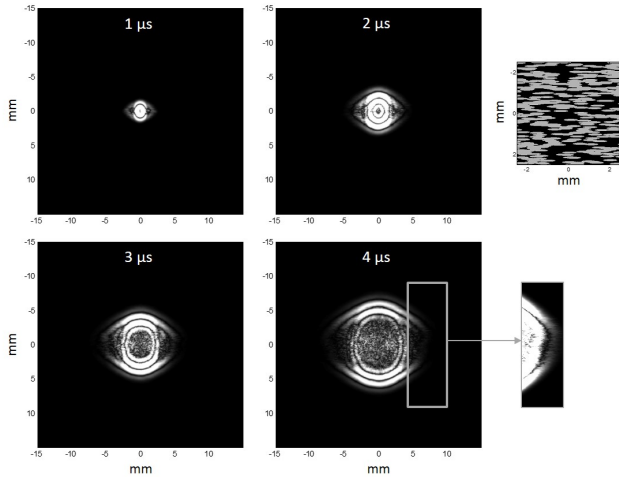


Figure 5: Estimation of the coherent wave propagation from a source point in a 2D anisotropic porous media with a 50% solid fraction. Left: 4 snapshots, taken at $1 \mu s$, $2 \mu s$, $3 \mu s$ and $4 \mu s$. Top right: detail of one realization showing the direction of the ellipses. Bottom right: A part of the fourth snapshot with a saturated contrast to better observe the fast wave. Two waves are observed for a propagation parallel to the main axis, only one is observed for a perpendicular propagation.

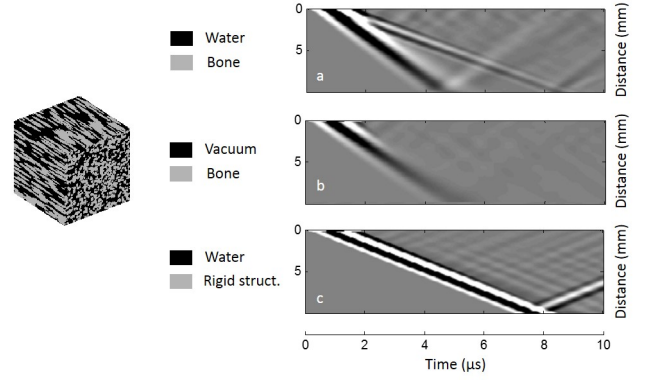


Figure 6: Left: The same 3D arrangement (collection of solid ellipsoidal scatterers, with a 50% solid fraction, propagation along the main direction) was used for the three simulations. Grey: solid phase. Black: fluid. (a) Reference simulation (same as in Fig.1a), Solid: Bone. Fluid: Water. Two wavefronts are clearly distinguished. (b) Solid: Bone. Fluid: Vacuum. Only one wavefront is observed, with a velocity (around $2.5 \text{ mm} \cdot \mu s^{-1}$) close to that of the fast wave (around $2.6 \text{ mm} \cdot \mu s^{-1}$) in (a). (c) Solid: Infinite density. Fluid: Water. Only one wave is observed, with a velocity (around $1.45 \text{ mm} \cdot \mu s^{-1}$) close to that of the slow wave (around $1.3 \text{ mm} \cdot \mu s^{-1}$) in (a)

404 actually be propagating with very close velocities. The nega-
 405 tive dispersion obtained from the propagation of a single wave,
 406 perpendicular to the main orientation of the scatterers could be
 407 re-interpreted in the light of this last result. If two waves are
 408 actually propagating with close velocities, the corresponding
 409 pulses could be interfering, leading to an apparent negative dis-
 410 persion, as observed by Anderson et al. [31].

4. Insights on the nature of the two waves

4.1. Limit cases

413 One great advantage of numerical simulations is the possi-
 414 bility to fully control the properties of the simulated medium.
 415 Fluid or solid properties were modified in the reference simula-
 416 tion, where two waves could be observed. First, water was repla-
 417 ced by vacuum (density and elastic constants were set equal to
 418 zero). Ultrasound propagation could therefore only occur in
 419 the solid frame. Second, in another simulation, the solid phase
 420 (which had initially the properties of bulk bone) was turned
 421 into a perfectly rigid frame, forcing the ultrasound propagation
 422 to occur only through water. Those two cases, which can
 423 be seen as limit cases when the density of the fluid (respec-
 424 tively solid) phase reaches zero (respectively infinity) would
 425 have been nearly impossible to study experimentally. The res-
 426 ults are compared with that of the reference case in Figure 6.

427 When ultrasound propagation was only allowed in one of the
 428 two phases, only one wavefront was observed, whereas there
 429 were clearly two distinguishable wavefronts in the reference
 430 simulation. Furthermore, the velocity of the wave when the propa-
 431 gation occurred only through the solid (respectively fluid)
 432 was close to that of the fast (respectively slow) wave in the
 433 reference case. This result suggests that the fast wave travels
 434 mostly through the solid frame, and the slow wave mostly
 435 through the fluid, in accordance with previous results [23]. It

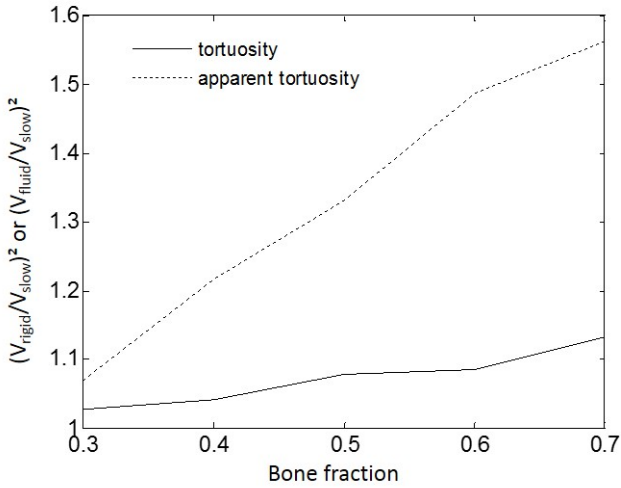


Figure 7: Tortuosity (deduced from rigid frame velocity) and apparent tortuosity (deduced from slow wave velocity) as a function of bone fraction

is interesting to notice that the speed V_{rigid} of the wave propagating only in water with a perfectly rigid frame (Fig.6c), around $1.45 \text{ mm} \cdot \mu\text{s}^{-1}$, is lower than the speed of sound in water ($V_{fluid} = 1.5 \text{ mm} \cdot \mu\text{s}^{-1}$) but larger than V_{slow} in Sec.3.2, around $1.3 \text{ mm} \cdot \mu\text{s}^{-1}$. These values gives interesting information. First, $V_{rigid} < V_{fluid}$ and $V_{slow} < V_{fluid}$ which accounts for the tortuosity of propagation paths in water. However, as $V_{rigid} \neq V_{slow}$ the slow wave formula (Eq.9) described in Sec.2.4 does not seem to hold. This suggests that the mechanical properties of the skeleton influence the slow wave velocity, which invalidates the stiff frame hypothesis. If tortuosity were to be estimated from velocity measurements, one should use V_{rigid} and Eq.9. Still, from an experimental and practical point of view, we could define an "apparent tortuosity" from Eq.9 using V_{slow} , which, unlike V_{rigid} , is accessible from real experiments. Figure 7 shows the tortuosity and "apparent tortuosity" obtained at various solid fractions, ranging from 30% to 70%. Tortuosity and "apparent tortuosity" are both confined between 1.03 and 1.56 and seem to converge to 1 at low solid fraction. The difference is at its highest for high solid fractions (low porosity). Hence, in the case of actual cancellous bones where porosity lies between 75% and 95%, the difference may not be so significant. This could open an interesting perspective: the possibility to measure tortuosity from the slow wave velocity in highly porous bones.

To conclude this subsection we confront the velocity measured in the reference simulation to the prediction of Eq.8. To do so we extract V_{dry}^L from the simulation where the fluid is turned into vacuum (Fig.6b) by a time-of-flight measurement. Then the only missing parameter is the geometric tortuosity α . As we have seen just before there are two different values for this parameter. The stiff frame assumption implies that ($K_{dry} \gg K_f$ and $N_{dry} \gg K_f$) with K_{dry} and N_{dry} the bulk and shear moduli of the dry sample [26]. These two parameters can be retrieved from $V_{dry}^L = 2.5 \text{ mm} \cdot \mu\text{s}^{-1}$ and to $V_{dry}^T = 0.8 \text{ mm} \cdot \mu\text{s}^{-1}$ i.e., the transverse speed of sound in the dry sample (see Appendix A) that we can also calculate thanks to

the propagation of a plane shear wave. We find $K_{dry} = 5.2 \text{ GPa}$ and $N_{dry} = 0.6 \text{ GPa}$, to be compared to $K_f = 2.25 \text{ GPa}$. This confirms that the stiff frame assumption does not hold, and explains why the apparent tortuosity differs from the actual tortuosity.

However, using the fast wave formula (Eq.8) and knowing that both values of tortuosity remain close to 1 and that ρ_s is almost twice ρ_f , one obtains:

$$\frac{\Phi_f \rho_f}{\Phi_s \rho_s} \left(1 - \frac{1}{\alpha}\right) \ll 1 \quad (11)$$

and as a consequence

$$V_{fast} = \frac{V_{dry}^L}{\sqrt{1 + \frac{\Phi_f \rho_f}{\Phi_s \rho_s} \left(1 - \frac{1}{\alpha}\right)}} \approx V_{dry}^L = 2.5 \text{ mm} \cdot \mu\text{s}^{-1} \quad (12)$$

This explains the observations made from figure 6, and agrees with the calculation with either one or the other value for the tortuosity:

$$V_{fast}^{tort} = 2.5 \text{ mm} \cdot \mu\text{s}^{-1} \quad (13)$$

$$V_{fast}^{app\ tort} = 2.3 \text{ mm} \cdot \mu\text{s}^{-1} \quad (14)$$

Both values are close to the actual measured fast wave velocity ($2.6 \text{ mm} \cdot \mu\text{s}^{-1}$ deduced from the value at 1 MHz in Fig.4).

Although the stiff frame assumption might not hold here, it appears that this does not affect the prediction of V_{fast} using Biot's theory. Biot's theory therefore gives correct orders of magnitude for the fast wave velocity in our anisotropic porous models, even beyond the frequency limit of this theory. It does not, however, predict the positive dispersion of the two waves, or the negative dispersion of the single wave, and does not account for scattering losses.

4.2. Phase shift of the fast and slow waves

One of the most striking results in Biot's model is that the fast and slow waves are associated respectively to in-phase and out-of-phase displacements of the fluid and solid skeleton. Interestingly, numerical simulation gives us the possibility to check if the average motion within the medium follows this peculiar behavior. As a last point of this paper, we studied the phase shifts between the displacements in each phase (solid or fluid) according to the method explained in Sec.2.4. Note that we have tested both to integrate particle velocities only at the interfaces or over the whole phases and it has shown very little difference, for both 70% and 50% solid fractions. As a consequence only the integrations over the whole phases are represented in Fig.8.

In each case, the phase shift was close to but not exactly that predicted by Biot: the observed phase shifts are 10° (70% solid fraction) and 19° (50% solid fraction) for the fast wave and respectively 174° and 163° for the slow wave. In a previous work [23] it was suggested that the physical origin for the occurrence of two waves could be that the bone trabeculae (or the connected ellipsoids) act as waveguides. This is the reason why we

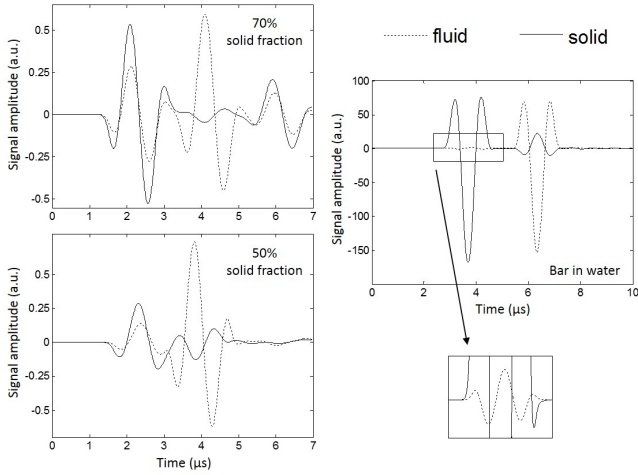


Figure 8: Left: particle velocity in the fluid and solid phases for a 70% (top) or a 50% (bottom) solid fraction sample taken at a 4mm propagation depth. Comparison with the particle velocity taken at a 8 mm propagation depth at each side of the interface of a solid bar in water (bottom right).

also studied the phase shifts between the motion of the fluid and solid phase in a very simple waveguide: a bar immersed in water. This particular case also exhibits two waves (Fig.8), with a guided wave in the bar re-radiating in water. Here too, the particle velocities are nearly in phase opposition for the slow wave (177° shift). However the shift for the fast wave is around 131° and so neither in nor out of phase. Though there is a discrepancy for the fast wave this result on a very simple case could be consistent with the fact that Biot's theory could be a particular, low frequency, case of a more general theory of the propagation of elastic waves in biphasic media involving guided waves.

5. Conclusion

The aim of this study was to compare quantitatively - in terms of velocities and attenuations - the results obtained from the simulation of elastic waves propagation in numerical anisotropic porous media with two theoretical approaches: Biot's model and a first-order multiple scattering model known as the Independent Scattering Approximation (ISA). The ISA was shown to provide good predictions of both velocity and attenuation coefficient when only one longitudinal wave occurred, and for low solid fractions (less than 10%). It was however unsuccessful to predict the two compressional waves occurring from the propagation along the main orientation of the scatterers. On the other hand, homogenization theories such as Biot's or Wood's were found in good agreement with the numerical results, whether one or two waves were observed, but only for the velocities and not for the attenuations. This suggests that attenuation is probably better predicted based on multiple scattering theories. Yet at higher solid fractions, the ISA will have to be replaced by higher-order approximations.

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Appendix A. Velocity of the fast wave

Under the stiff frame assumption, Johnson [26] gives simple relations for fast, slow and transverse waves velocities. In particular the fast and transverse waves velocities can be written

$$V_{fast} = \sqrt{\frac{K_{dry} + \frac{4}{3}N_{dry}}{\Phi_s\rho_s + \Phi_f\rho_f\left(1 - \frac{1}{\alpha}\right)}} \quad (\text{A.1})$$

$$V^T = \sqrt{\frac{N_{dry}}{\Phi_s\rho_s + \Phi_f\rho_f\left(1 - \frac{1}{\alpha}\right)}} \quad (\text{A.2})$$

where K_{dry} and N_{dry} are the bulk and shear moduli of the dry sample (no filling fluid). When fluid is present, the stiff frame hypothesis is fulfilled as long as $K_{dry} \gg K_f$ and $N_{dry} \gg K_f$, with K_f the bulk modulus of the fluid. In our case K_{dry} and N_{dry} were deduced from transverse V_{dry}^T and longitudinal V_{dry}^L velocities in the dry sample ($\rho_f = 0$) as follows:

$$V_{dry}^L = \sqrt{\frac{K_{dry} + \frac{4}{3}N_{dry}}{\Phi_s\rho_s}} \quad (\text{A.3})$$

$$V_{dry}^T = \sqrt{\frac{N_{dry}}{\Phi_s\rho_s}} \quad (\text{A.4})$$

As a consequence

$$N_{dry} = \Phi_s\rho_s V_{dry}^T{}^2 \quad (\text{A.5})$$

$$K_{dry} = \Phi_s\rho_s \left(V_{dry}^L{}^2 - \frac{4}{3}V_{dry}^T{}^2 \right) \quad (\text{A.6})$$

which means that Eq.A.1 can be advantageously simplified as in Sec.2.4

$$\begin{aligned} V_{fast} &= \sqrt{\frac{\Phi_s\rho_s V_{dry}^L{}^2}{\Phi_s\rho_s + \Phi_f\rho_f\left(1 - \frac{1}{\alpha}\right)}} \\ &= \frac{V_{dry}^L}{\sqrt{1 + \frac{\Phi_f\rho_f}{\Phi_s\rho_s}\left(1 - \frac{1}{\alpha}\right)}} \end{aligned} \quad (\text{A.7})$$

It is interesting to notice that according to this equation, the fast wave velocity does not depend on the transverse wave velocity, which was not so clear while looking at Eq.A.1.

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