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To cite this version:
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Abstract

Nonlinear mixed effect models (NLMEM) with multiple responses are increasingly used in pharmacometrics, one of the main examples being the joint analysis of the pharmacokinetics (PK) and pharmacodynamics (PD) of a drug. Efficient tools for design evaluation and optimisation in NLMEM are necessary. The R functions PFIM 1.2 and PFIMOPT 1.0 were proposed for these purposes, but accommodate only single response models. The methodology used is based on the Fisher information matrix, developed using a linearisation of the model. In this paper, we present an extended version, PFIM 3.0, dedicated to both design evaluation and optimisation for multiple response models, using a similar method as for single response models. In addition to handling multiple response models, several features have been integrated into PFIM 3.0 for model specification and optimisation. The extension includes a library of classical analytical pharmacokinetics models and allows the user to describe more complex models using differential equations. Regarding the optimisation algorithm, an alternative to the Simplex algorithm has been implemented, the Fedorov-Wynn algorithm to optimise more practical D-optimal design. Indeed, this algorithm optimises design among a set of sampling times specified by the user. This R function is freely available at www.pfim.biostat.fr. The efficiency of this approach and the simplicity of use of PFIM 3.0 are illustrated with a real example of the joint PKPD analysis of warfarin, an oral anticoagulant, with a model defined by ordinary differential equations.

Key words: Fisher information matrix; nonlinear mixed effect models; multiple response models; optimal designs; D-optimality; PFIM.
1. Introduction

Nonlinear mixed effect models (NLMEMs) are increasingly used for the analysis of longitudinal data describing a biological process. They allow the estimation of the mean value of parameters in population studies and their inter-individual variability. They are also used for the joint modelling of several biological responses, such as the joint analysis of pharmacokinetic (PK) and pharmacodynamic (PD) data [1]. Pharmacokinetics deals with the time-course of drug concentration, whereas pharmacodynamics refers to the time-course of drug action in the body. In pharmacometrics, analysis through a NLMEM is called the population approach. To estimate parameters in NLMEMs, maximum likelihood estimation is used primarily, although the likelihood for these models has no analytical solution. Specific algorithms, implemented in several software packages, have therefore been proposed to perform this maximisation [2].

Before the estimation step, the investigator of a study is confronted with the choice of the design which is crucial for an efficient estimation of model parameters. A design in NLMEM, also called a population design, is composed of the number of elementary designs (or groups) and the specification of each elementary design and the associated number of subjects. In this setting, the term elementary design is used to describe a collection of subjects that have identical design characteristics defined by the number of sampling times and their allocation in time. To evaluate and compare population designs, a statistical approach based on the theory of optimum experimental design in classical nonlinear models and described for instance by Atkinson and Donev [3] or by Walter and Pronzato [4] has been extended to NLMEMs. This theory is based on the Fisher information matrix, whose inverse, according to the Cramer-Rao inequality, is the lower bound of the variance covariance matrix of any unbiased estimators of the parameters. Due to the lack of an analytical expression for the likelihood in NLMEM, an exact expression of the Fisher information matrix cannot be
defined. That is why an expression based on a linearisation of the model around the expectation of random effects has been proposed by Mentré et al. [5] and extended by Retout and Mentré [6] in the context of a single response model. The usefulness of this approach has been demonstrated, both by simulation [7, 8] and in real pharmacokinetic studies [9, 10]. To make the procedure more accessible to investigators, the approximate expression of the Fisher information matrix has been implemented in the R function PFIM 1.2, which can be used for design evaluation and comparison [11].

Regarding optimisation, two approaches can be used, either the optimisation of exact designs or of statistical designs. In the case of optimisation of exact designs, the group structure of the design is fixed: the number of elementary designs, the number of samples per design and the number of subjects per elementary design are given and the design variables used to optimise are only the sampling times. Optimisation of statistical designs consists in optimising both the allocation of the sampling times and the whole group structure, that is to say the number of elementary designs, the number of samples per elementary designs and the proportion of subjects in each elementary design. An exact design is then derived by rounding off the proportion of subjects in each elementary design. Optimisation based on the D-optimality criterion has been implemented in the R function PFIMOPT 1.0 [12], where the optimal design is the one that maximises the determinant of the Fisher information matrix. In PFIMOPT 1.0, both exact or statistical optimisation can be performed using the general Simplex algorithm [13]. It optimises the sampling times in given continuous intervals.

The main limitation of PFIM 1.2 and PFIMOPT 1.0 is that both aim at evaluating and optimising population designs only for single response models. Moreover, the model had to be written using an analytic expression and thus the need to use more complex models is limited. Regarding optimisation, the Simplex algorithm is a general optimisation algorithm. However, even if its applicability has been shown in pharmacokinetic examples [8], when
there are a large number of parameters to optimise, or when the model is complex, it could have difficulties in converging towards the optimal design and should sometimes be run again using the optimised design as a new initial design.

Recently, the expression for the Fisher information matrix has been extended in the context of a joint estimation of a set of parameters from multiple responses using the same linearisation as for single response [14-16]. In that context, some parameters can be included in several responses, as for instance in a classical PKPD model with the PD response depending on the PK parameters. That joined estimation is then more informative, obtaining information on the PK parameters from both PK and PD responses. However, it increases the complexity of the computation of the Fisher information matrix compared to its computation for each response (single response). The relevance of the use of this approximation of the Fisher information matrix for that multiple response NLMEM context has been shown by Bazzoli et al. [14] through simulation of a PKPD model; results were very similar to those obtained from a more exact computation of the information matrix, without any linearization, using stochastic approximation through the SAEM algorithm of MONOLIX [17].

This expression has been implemented in PFIM 3.0, an extension of PFIM to handle both design evaluation and optimisation in NLMEMs with multiple responses. The extension includes a library of classical analytical pharmacokinetics models and allows more complex models to be described using a system of differential equations. Regarding the optimisation step, an alternative to the Simplex has been added in PFIM 3.0, the Fedorov-Wynn algorithm. It is a specific design optimisation algorithm implemented in PFIM 3.0 for statistical optimisation [5, 18] which has the property of converging towards the D-optimal design.

The aim of this paper is to present PFIM 3.0. In section 2, we present a nonlinear mixed effect multiple response design and model and the expression of the population Fisher information matrix for multiple responses. Then, the structure of PFIM 3.0 and the description
of its features and use are presented in Section 3. Lastly, an example of the use of PFIM 3.0 to design a new trial for the joint analysis of the pharmacokinetics and pharmacodynamics of warfarin, an oral anticoagulant, is provided in Section 4.

2. Statistical methods

a. Nonlinear mixed effect multiple response design and model

A design for NLMEM, i.e. a population design, is composed of \( N \) individuals to whom we allocate an “elementary” design \( \xi_i, i = 1, \ldots, N \). Each elementary design is defined by a number \( n_i \) of sampling times and their allocation in time. A population design is therefore described by \( N \) elementary designs:

\[
\Xi = \{\xi_1, \ldots, \xi_N\} \tag{1}
\]

leading to a total number \( n \) of observations.

In the case of a multiple response model, an elementary design for one individual \( i \) is composed of several sub-designs, i.e. \( \xi_i = (\xi_{i1}, \xi_{i2}, \ldots, \xi_{ik}) \), where \( \xi_{ik} \) is the design associated with the \( k \)th response, \( k = 1, \ldots, K \). \( \xi_{ik} \) is defined by \( (t_{i1k}, t_{i2k}, \ldots, t_{in_k}) \), the vector of the \( n_k \) sampling times for the observations of the \( k \)th response, so that \( n_i = \sum_{k=1}^{K} n_{ik} \).

Usually, population designs are composed of a limited number \( Q \) of groups of individuals. Each group is defined by an elementary design \( \xi_q, q = 1, \ldots, Q \), which is composed, for the \( k \)th response, of \( n_{qk} \) sampling times \( (t_{q1k}, t_{q2k}, \ldots, t_{qnk}) \) to be performed on a number \( N_q \) of individuals. The population design can then be noted as follows:

\[
\Xi = \{(\xi_1, N_1); (\xi_2, N_2); \ldots; (\xi_Q, N_Q)\} \tag{2}
\]
A nonlinear mixed effects multiple response model or a multiple response population model is defined as follows. The vector of observations $Y_i$ for the $i^{th}$ individual is defined as the vector of the $K$ different responses:

$$Y_i = [y_{i1}^T, y_{i2}^T, \ldots, y_{ik}^T]^T$$ (3)

where $y_{ik}, \ k = 1, \ldots, K$, is the vector of observations for the $k^{th}$ response. Each of these responses is associated with a known function $f_k$ which defines the nonlinear structural model. The $K$ functions $f_k$ can be grouped in a vector of multiple response models $F$, such as:

$$F(\theta_i, \xi_i) = [f_1(\theta_i, \xi_{i1})^T, f_2(\theta_i, \xi_{i2})^T, \ldots, f_K(\theta_i, \xi_{ik})^T]^T$$ (4)

where $\theta_i$ is the vector of all the individual parameters needed for all the response models for individual $i$. The vector of individual parameters $\theta_i$ depends on $\beta$, the $p$-vector of the fixed effects parameters and on $b_i$ the vector of the $p$ random effects for individual $i$. The relation between $\theta_i$ and $(\beta, b_i)$ is modelled by a function $g$, $\theta_i = g(\beta, b_i)$, which is usually additive, so that $\theta_i = \beta + b_i$, or exponential so that $\theta_i = \beta \exp(b_i)$. It is assumed that $b_i \sim N(0, \Omega)$ with $\Omega$ defined as a $p \times p$-diagonal matrix, each diagonal element $\omega^2_r, r = 1, \ldots, p$, representing the variance of the $r^{th}$ component of the vector $b_i$. We consider here only the case of a diagonal $\Omega$ matrix with no correlation between the random effects.

The statistical model is thus given by:

$$Y_i = F\left(g(\beta, b_i), \xi_i\right) + \epsilon_i$$ (5)

where $\epsilon_i$ is the vector composed of the $K$ vectors of residual errors $\epsilon_{ik}, \ k = 1, \ldots, K$, associated with the $K$ responses. We also suppose $\epsilon_{ik} \sim N\left(0, \Sigma_{ik}\right)$ with $\Sigma_{ik}$ a $n_{ik} \times n_{ik}$-diagonal matrix such that
\[
\Sigma_{ik} \left( \beta, b_i, \sigma_{\text{int}}^k, \sigma_{\text{slope}}^k, \xi_{ik} \right) = \text{diag} \left( \sigma_{\text{int}}^k + \sigma_{\text{slope}}^k f_k \left( g \left( \beta, b_i \right), \xi_{ik} \right) \right)^2
\]  

where \( \sigma_{\text{int}}^k \) and \( \sigma_{\text{slope}}^k \) qualify the model for the variance of the residual error of the \( k^{th} \) response. The case \( \sigma_{\text{slope}}^k = 0 \) returns a homoscedastic error model, whereas the case \( \sigma_{\text{int}}^k = 0 \) returns a constant coefficient of variation error model. The general case where the two parameters differ from 0 is called a combined error model. We then note \( \Sigma_i \left( \beta, b_i, \sigma_{\text{int}}^i, \sigma_{\text{slope}}^i, \xi_i \right) \) the variance of \( \xi_i \), over the \( K \) responses, such that \( \Sigma_i \) is a \( n_i \times n_i \)-diagonal matrix composed of each diagonal element of \( \Sigma_{ik} \) with \( k = 1, \ldots, K \). \( \sigma_{\text{slope}}^k \) and \( \sigma_{\text{int}}^k \) are two vectors of the \( K \) components \( \sigma_{\text{int}}^k \) and \( \sigma_{\text{slope}}^k \), \( k = 1, \ldots, K \), respectively. Finally, depending on the value of \( b_i \), we assume that the errors \( \xi_i \) are independently distributed.

Let \( \Psi \) be the vector of population parameters to be estimated, i.e.

\[
\Psi^T = (\beta^T, \omega_i^2, \ldots, \omega_p^2, \sigma_{\text{int}}^T, \sigma_{\text{slope}}^T)
\]

and let \( \lambda \) be the vector of variance terms \( \lambda^T = (\omega_i^2, \ldots, \omega_p^2, \sigma_{\text{int}}^T, \sigma_{\text{slope}}^T) \), so that \( \Psi^T = (\beta^T, \lambda^T) \).

**b. Fisher information matrix**

The population Fisher information matrix \( M_f (\Psi, \Xi) \) for multiple response models with the population design \( \Xi \) is given by:

\[
M_f (\Psi, \Xi) = E \left( \frac{\partial^2 L(\Psi, Y)}{\partial \Psi \partial \Psi^T} \right)
\]  

where \( L(\Psi, Y) \) is the log-likelihood of the vector of the whole observations \( Y \) for the population parameters \( \Psi \).

Assuming independence across individuals, this log-likelihood can be defined as the sum of the log-likelihoods of the vectors of observations for each individual:
\[ L(\Psi, Y) = \sum_{i=1}^{N} L(\Psi, Y_i) \]. Therefore, taking the second derivatives, the population Fisher information matrix for \( N \) individuals can also be defined as the sum of the \( N \) elementary information matrices \( M_F(\Psi, \xi_i) \) computed for each individual \( i \):

\[ M_F(\Psi, \Xi) = \sum_{i=1}^{N} M_F(\Psi, \xi_i) \tag{8} \]

In the case of a limited number \( Q \) of groups, it is expressed as:

\[ M_F(\Psi, \Xi) = \sum_{q=1}^{Q} N_q M_F(\Psi, \xi_q) \tag{9} \]

The expression of an elementary Fisher information matrix for multiple responses has been described in Bazzoli et al. [14] where further details are given. For the sake of simplicity, we have omitted the index \( i \) for the individual in the following. The Fisher information matrix is a block matrix depending on the approximated marginal expectation \( E \) and variance \( V \) of the observations.

\[ M_F(\Psi, \xi) \equiv \frac{1}{2} \begin{pmatrix} A(E,V) & C(E,V) \\ C^T(E,V) & B(E,V) \end{pmatrix} \tag{10} \]

where

\[ \begin{align*}
(A(E,V))_{ml} &= 2 \frac{\partial E^T}{\partial \beta_m} V^{-1} \frac{\partial E}{\partial \beta_l} + \text{tr} \left( \frac{\partial V}{\partial \beta_m} V^{-1} \frac{\partial V}{\partial \beta_m} V^{-1} \right) \text{ with } m \text{ and } l = 1, \ldots, p \\
(B(E,V))_{ml} &= \text{tr} \left( \frac{\partial V}{\partial \lambda_m} V^{-1} \frac{\partial V}{\partial \lambda_l} V^{-1} \right) \text{ with } m \text{ and } l = 1, \ldots, \text{dim}(\lambda) \\
(C(E,V))_{ml} &= \text{tr} \left( \frac{\partial V}{\partial \lambda_l} V^{-1} \frac{\partial V}{\partial \beta_m} V^{-1} \right) \text{ with } l = 1, \ldots, \text{dim}(\lambda) \text{ and } m = 1, \ldots, p 
\end{align*} \]
Note that in this work, we consider that the variance of the observations with respect to the mean parameters is constant, i.e. $C(E, V)_{ml} = 0$ and $A(E, V)_{ml} = 2 \frac{\partial E^T}{\partial \beta_m} V^{-1} \frac{\partial E}{\partial \beta_l}$.

3. Implementation of PFIM 3.0 using R software

The development of the population Fisher information matrix described previously has been implemented in an extension of the R function PFIM: PFIM 3.0. It allows a design evaluation and optimisation of multiple response population models. PFIM 3.0 can be used for single response models instead of the previous version PFIM 1.2 or PFIMOPT 1.0. In addition to the extension for multiple response models, options have been added for model specification and for design optimisation. The free statistical R software (R 2.4.1 and higher versions) [19] (http://cran.r-project.org/) is required to use PFIM 3.0. This function is freely available on the PFIM website http://www.pfim.biostat.fr as well as an extensive documentation and some examples to use it.

a. Model specification

In PFIM 3.0, the model has to be entered in a model file called by default model.r. It must be specified by the user either in its analytical form or by using a system of differential equations. This extension to a system of differential equations requires the use of the lsoda function included in the library “odesolve” implemented by Setzer to solve the system [20] and the fdHess function included in the library “nlme” developed by Pinheiro and Bates [21] for numerical derivatives. The lsoda function uses a function of the same name written in Fortran by Petzold and Hindmarsh [22, 23]. This function solves a system of differential
equations using the Adams method, a predictor–corrector method for non-stiff systems; it uses
the BDF (Backward Differentiation Formula) for stiff systems. Regarding the function
\textit{fdHess}, it evaluates an approximate gradient of a scalar function using finite differences.

Moreover, a library of standard PK models has been added. Twelve PK models are
included: one or two compartment models, with first order oral, bolus or infusion
administration and after a single dose, multiple doses or steady-state. Only models with first
order elimination are presently available in the library of PFIM 3.0. Currently, there are no
models with lag time. A list of the models and their characteristics included in the library is
given in Table 1.

\textit{b. Optimisation algorithms}

An alternative to the Simplex algorithm has been added in PFIM 3.0, the Fedorov-Wynn
algorithm. It is specifically dedicated to design optimisation problems and has the property of
convergence towards the D-optimal design [5, 24, 25]. It optimises statistical designs (i.e the
group structure, the number of samples per subject and the sampling times) for a given total
number of samples. Minimum and maximum number of samples per subject are specified. To
optimise the design, it considers only a set of possible sampling times defined by the user.
This can be an advantage in clinical practice to avoid unfeasible sampling times. Design
optimisation can be constrained to the same sampling times across responses or not. It is also
possible to give each elementary design sampling times from a different set of finite sampling
times. For instance, it is possible to specify that each elementary design should include two
points among a first set of sampling times (for instance Day 1) and a third point among a
second set of sampling times (for instance Day 2). The user therefore specifies a list of
sampling windows, and for each window, the set of possible sampling times and the minimum
and maximum number of sampling times within that window. The Fedorov-Wynn algorithm
is programmed in C code and is linked to PFIM 3.0 through a dynamic library. Moreover, PFIM 3.0 uses the function `combn` in the R package “combinat” to generate all possible elementary designs from the set of pre-specified sampling times. To start the algorithm, an initial population design is required.

c. Input file

The inputs to PFIM 3.0 are entered in R objects by filling in one input file called by default `stdin.r`. The same function PFIM 3.0 is used for the evaluation and optimisation of population designs, which is why the user has to notify it in the input file in a specific R object. Then, this input file is composed of two main sections of R objects. The first one is composed of general R objects required for evaluation and optimisation such as the model form (analytical or differential equation system), the parameterisation of the structural model, the type of inter-individual random effect model (additive or exponential), values of $\sigma_{\text{inter}}$ and $\sigma_{\text{slope}}$ for the variance error model and a priori values of the fixed effect parameters and variance parameters. The user also specifies the population design to evaluate or, for optimisation, an initial population design. They are defined as a group of elementary designs each one associated with a number of subjects. The second section is dedicated to design optimisation where the specific objects are algorithm options. First, the user specifies if the same sampling times are required for each response or not. Then, the algorithm is chosen (Simplex or Fedorov-Wynn). If the Fedorov-Wynn is required, the user must provide the following information: the number of sampling windows, the list of the allowed sampling times for each sampling window, the list of the number of allowed sampling times for each sampling window, the maximum and minimum total number of sampling times per subject.
Graphical representations are provided automatically if graph objects are completed. Once the model file and the input file are filled in, the user can run the function PFIM 3.0 by calling the function in the R Console window: PFIM().

d. Output file

The results are written to an output file named by default stdout.r or with a name specified in the input file. PFIM 3.0 returns a summary of the input, the population design evaluated or optimised, the associated population Fisher information matrix, the standard error expected on each parameter, the corresponding coefficient of variation (%), its determinant and a criterion $\Phi$ defined as the determinant standardized by the dimension of the vector $\Psi$: $\Phi(\Xi) = \left[\det\left(M(\Psi, \Xi)\right)\right]^{-\frac{1}{\dim(\Psi)}}$. When design optimisation is performed, the list of the algorithm options is also specified in the output file. For instance, for the Fedorov-Wynn algorithm, the set of allowed sampling times, the number of sampling times, the maximum and the minimum number of points for each response are returned. The optimised design is reported with the proportion of subjects optimised by the Fedorov-Wynn algorithm and their absolute number. If a graph has been supplied in the input file, the mean profile of each response and the associated sampling times of the evaluated or optimised design are represented on an R graph.

4. Illustration of design for PKPD of warfarin

We aim to illustrate the use of PFIM 3.0 for the joint PKPD modelling of the time course of total racemic warfarin plasma concentrations and effect on prothrombin complex activity (PCA), by designing a new trial for this joint population analysis.

To do this, we use an example extracted from a classical pharmacology study published 40 years ago, by O’Reilly et al. [26, 27] on the pharmacokinetics and pharmacodynamics of
warfarin. Previously, these data have been fitted by Holford for the analysis of the dose-effect relationship of warfarin [28] using nonlinear mixed effects models. This example was presented in [29] but using different constraints in the optimisation step.

a. Population design and model

Warfarin is administered by a single oral dose of 100 mg. A one compartment model with first order absorption and elimination adequately describes the concentration data (PK). The PCA effect is described by a turnover model with inhibition of the input (PD). The model can be written as a system of ordinary differential equations:

\[
\begin{align*}
\frac{df_{PK}(\theta_{PK}, t)}{dt} &= k_a \cdot D \cdot e^{-k_at} \cdot \frac{CL}{V} \cdot f_{PK}(\theta_{PK}, t) \\
\frac{df_{PD}(\theta_{PD}, t)}{dt} &= R_{in} \left( 1 - \frac{f_{PK}(\theta_{PK}, t)}{IC_{50} + f_{PK}(\theta_{PK}, t)} \right) \cdot -k_{out} \cdot k \cdot f_{PD}(\theta_{PD}, t)
\end{align*}
\]

(11) (12)

where \( f_{PK} \) and \( f_{PD} \) are respectively the warfarin plasma concentrations and effect model and \( \theta_{PK} \) and \( \theta_{PD} \) the parameters needed for each response. D is the dose of warfarin. The vector of the PK parameters is \( \theta_{PK} = (k_a, CL, V) \) with \( k_a \) the absorption constant, \( CL \) the apparent clearance and \( V \) the apparent volume of distribution. PCA is controlled by the rate of input \( R_{in} \) and the output rate constant \( k_{out} \) of the response, and its baseline value is given by the ratio of those parameters \( R_{in}/k_{out} \). The vector of PD parameters is thus defined by \( \theta_{PD} = (R_{in}, IC_{50}, k_{out}) \) where \( IC_{50} \) is the drug concentration which produces 50% of maximum inhibition achieved at the effect site.

The inter-individual random effects are assumed to be exponential for all parameters. We associate a proportional error for the PK model and an additive error for the PD model characterised by the parameters \( \sigma_{slopePK} \) and \( \sigma_{interPD} \). Thus, the vector of population
parameters $\Psi$ is described by the vector of fixed effects $\beta^T = (\beta_{\text{IC}} , \beta_{\text{CL}} , \beta_{\text{V}} , \beta_{R} , \beta_{\text{IC} \text{int}} , \beta_{\text{IC} \text{int}})$ and by $\lambda^T$ the vector composed of the variance of the random effects and of the parameters for the error models such that $\lambda^T = (\alpha^2_{\text{IC}} , \alpha^2_{\text{CL}} , \alpha^2_{\text{V}} , \alpha^2_{R} , \alpha^2_{\text{IC} \text{int}} , \alpha^2_{\text{IC} \text{int}} , \sigma_{\text{slope PK}} , \sigma_{\text{int } \text{PD}})$.

Values of the parameters are those obtained by Holford performing the simultaneous PKPD population analysis of warfarin. The population parameters were estimated by maximum likelihood, with the NONMEM software [30]. They are given in Table 2.

**b. Design optimisation for a new study**

Using PFIM3.0, we first evaluate the design used in the previous study to determine the joint population PKPD model. We call this design the “empirical design” denoted $\Xi_{\text{emp}}$. It involves one group of 32 healthy volunteers, with 13 sampling times at 0.5, 1, 2, 3, 6, 9, 12, 24, 36, 48, 72, 96, 120 hours for warfarin concentration measurements and 8 sampling times at 0, 24, 36, 48, 72, 96, 120, 144 hours for PCA measurement. $\Xi_{\text{emp}}$ thus has a total of 672 measurements.

We then optimise two population designs under several constraints using the Fedorov-Wynn algorithm implemented in PFIM 3.0. To do that, we first optimise a population design $\Xi_{\text{opt, iden}}$ with only four sampling times per elementary design common to both responses. This optimisation step is performed on the same number of 32 volunteers and the set of admissible sampling times is the set of times used in the empirical design except times 1 and 3 for PK. We then optimise a population design $\Xi_{\text{opt, diff}}$, with four sampling times per elementary design, but now, the optimal sampling times can be different for each response.

**c. Results**
Fig. 1 reports the model file used for the design evaluation and optimisation of the joint PKPD model of warfarin. The input file of PFIM 3.0 for the evaluation of $\Xi_{\text{emp}}$ is reported in Fig. 2. The output file of PFIM 3.0 for the optimal design $\Xi_{\text{opt._iden}}$ and the associated R graph output are represented in Fig. 3 and Fig. 4, respectively. The optimal designs with their associated criterion are reported in Table 3. $\Xi_{\text{emp}}$ and both optimal designs ($\Xi_{\text{opt._iden}}, \Xi_{\text{opt._diff}}$) are different by their group structure. $\Xi_{\text{opt._iden}}$ and $\Xi_{\text{opt._diff}}$ are composed of two elementary designs compared to only one for $\Xi_{\text{emp}}$. The Fedorov-Wynn algorithm optimises proportions of subjects, to be practical those numbers are then rounded to the nearest integers. Each elementary design involves 22 and 10 subjects for $\Xi_{\text{opt._iden}}$ and 29 and 3 for $\Xi_{\text{opt._diff}}$. In order to get more practical designs, $\Xi_{\text{opt._diff}}$ is simplified in a new design $\Xi_{\text{opt._diff simp}}$, composed of one group of 32 subjects, meaning that the 3 subjects originally included in the second elementary design have been added to the first elementary design. The two optimised designs have a smaller number of samples than the empirical design and are therefore less efficient with lower criterion values (Table 3). These two designs include a sample at 144 hours for the PK measurement whereas this sample is not present in the empirical design. Indeed, in population PK studies, when a proportional residual error is associated with the model, the optimal design includes the latest time among those proposed. In our case, for optimisation, we allowed the sample at 144 hours both for PK and PD responses.

In order to get an efficiency similar to that obtained with $\Xi_{\text{emp}}$, we then derived two “enlarged” designs $\Xi_{\text{opt._iden enl}}$ and $\Xi_{\text{opt._diff simp enl}}$ from the two optimal designs by increasing the number of subjects. 55 subjects and 52 subjects are involved in $\Xi_{\text{opt._iden enl}}$ and $\Xi_{\text{opt._diff simp enl}}$, meaning a total of number of sampling times of 440 and 416, respectively. The expected relative standard errors (RSE) of $\Xi_{\text{emp}}$, $\Xi_{\text{opt._iden enl}}$ and $\Xi_{\text{opt._diff simp enl}}$, i.e
standard error divided by the true value of the parameter, their total number of sampling times and the associated criterion are given in Table 4.

The evaluation of $\Xi_{\text{emp}}$ returns expected RSEs smaller than 30%, except for the variance parameters $\omega_{\text{IC}_w}^2$ and $\omega_{\text{res}}^2$, where they are rather large (about 68% and 41%) due to the small inter-subject variability of these parameters. Comparison between both “enlarged” designs shows that only the coefficient of variation of the estimation of the variance parameter $\omega_{\text{IC}_w}^2$ is increased compared to $\Xi_{\text{emp}}$. The expected RSE on the terms for residual errors increase in both optimal designs, but they remain lower than 10%. For other parameters, the results from design optimisation, in terms of precision of estimation, are very close to those obtained with the empirical design, although “enlarged” designs involve only 440 and 416 measurements, respectively, 35% and 38% fewer than the empirical design. This sample reduction, together with a similar efficiency, leads to a more ethical and less costly study. Using PFIM 3.0, we find new efficient designs to study the joint PKPD of warfarin fitting to different clinical constraints.

5. Discussion

In this paper, we consider the R function PFIM 3.0, an extension of the R functions PFIM 1.2 and PFIMOPT 1.0, allowing evaluation and optimisation of designs for nonlinear mixed effect models with multiple responses. The methodology is based on the Fisher information matrix, developed using a linearisation of the model, as proposed for a single response by Mentré et al. [5] and extended for multiple responses [14-16]. In addition to handling multiple response models, several features have been integrated in PFIM 3.0. First, regarding the model specification, it can now be specified as a system of differential equations; this is an alternative to analytical model specification, particularly useful for complex models. For
“classical” pharmacokinetic models, a library of twelve pharmacokinetic models is available. Concerning the optimisation algorithm, the Fedorov-Wynn algorithm has been added as an alternative to the general Simplex algorithm to optimise more practical D-optimal designs [24, 25]. Indeed, this algorithm allows design optimisation with fixed sampling times specified by the user which can be a great advantage with respect to medical constraints. This algorithm optimises the sampling times, which can be different or imposed to be identical across responses, but also optimises the group structure, using the approach of statistical designs. In addition to convergence issues using the Simplex algorithm compared to the Fedorov-Wynn [18], optimisation within continuous intervals of times can be more questionable: the Simplex algorithm can, sometimes for a long time, pursue the optimisation by exchanging at each iteration one or several sampling times by some others which differ from the previous by only few seconds. This greatly slows down the whole optimisation process and has no relevance in clinical practice.

We illustrate the different options of this new extension of PFIM using a real example of the simultaneous population analysis of the time course of warfarin concentration and its effect on the prothrombin complex activity after single dose administration. We optimise and derive two population designs using the Fedorov-Wynn algorithm, either imposing the same sampling times across responses or not, and we compare them to an “empirical design” used in the previous study of warfarin [26, 27]. Globally, the expected precision of estimations are in the same range for the three designs, even if the number of samples per patient, and thus, the total number of samples are considerably reduced (38% fewer samples in the optimised designs compare to the empirical one). In this example, we show the interest of the optimisation procedure implemented in PFIM 3.0 from a medical, economic as well as ethical point of view. This approach can also be used for other complex modelling, such as the pharmacokinetics of a parent drug and its metabolite [31]. However, in some situations, due to
the different time scales of the pharmacokinetic profiles between the parent drug and the metabolite, different sampling times for the two responses may be required. It should be noted however, using the Fedorov-Wynn algorithm under this assumption, the number of possible elementary designs will be large and can considerably increase the computation time and the memory requirement. Limiting the number of allowed sampling times for each sampling window also helps to avoid computational issues.

In the example, the prothrombin complex activity is controlled by the rate of input $R_{in}$ and the output rate constant $k_{out}$ of the response, and the initial condition is given by the ratio of these parameters $R_{in}/k_{out}$. It would be possible by using a modification of the parameterisation of the model to estimate the initial condition and $R_{in}$ or $k_{out}$ or even to fix the initial condition (for instance to 100%) and to estimate only $R_{in}$ or $k_{out}$.

The present development of the Fisher information matrix for multiple response models takes into account all the population parameters to be estimated. Another approach would have been to optimise separately one design for the PK parameters and one for the PD parameters, fixing the PK parameters to their mean in the population (sequential approach). However, this approach can be sub-optimal as no variability on the PK parameters is taken into account in the PD model. For instance, we compared this sequential approach to the one used in this paper, using the same example and similar constraints. We obtained the same optimal design for the PK model. Regarding the conditional D-optimal design for the PD, the design is quite different from $\Xi_{opt\_diff\_simp}$. Indeed, it involves two elementary designs with sampling times at (0, 24, 72, 96) and (0, 24, 36, 96) hours, and with 21 and 11 subjects, respectively. Comparison of the relative standard errors shows an increase by at least a factor 2 with this design compared to those obtained with $\Xi_{opt\_diff\_simp}$.

Following the first theoretical work on optimal design for NLMEM, this research theme has grown rapidly both in methodological and application developments. An email mailing
list server PopDesign is now available for questions and comments about the experimental
design of any study for which nonlinear mixed effect modelling is proposed for the analysis.
Since the first tool PFIM [11], there are now several different software tools that implement
an evaluation of the Fisher information matrix for population PK and PD models and propose
optimisation of the experimental designs (PFIM, PopDes, PopED, WinPOPT) [32]. All
software packages have ongoing development and have specific features according to the
needs and goals of the research teams that developed them. PFIM is the only one using the R
software.

PFIM 3.0 uses an approximate Fisher information matrix to evaluate and optimise design.
This approximation, using a linearisation of the model, may be a limitation. Indeed, even if
the relevance of this approximation has been shown by comparison to an exact Fisher
information matrix using a simulation study, the PKPD model used was very simple [14] and
may not be generalised to any complexity of the models. On the other hand, use of more
exact methods can be computationally intensive and thus time consuming. Their need is thus
questionable for nonlinear models for which design evaluation and optimisation require some
\textit{a priori} values of the parameters to be estimated, which are often not known precisely.

In parallel to PFIM 3.0, PFIM Interface 2.1, a graphical user interface version of PFIM 1.2
and PFIMOPT 1.0 has been recently proposed, allowing both design evaluation and
optimisation but only for single response models. PFIM Interface 2.1 has the same features
for model specification and optimisation algorithm as in PFIM 3.0. PFIM Interface will soon
be extended to accommodate multiple response models.

In addition to the new features added in PFIM 3.0, several other extensions are planned.
First, the current library includes only pharmacokinetic models. Extensions to the usual
pharmacokinetic-pharmacodynamic models are planned, even if model specification for
multiple responses is quite simple for the user as shown in the illustration using a system of
differential equations.

Then, the next version could integrate the computation of the Fisher matrix for a non-
diagonal variance of the random effects, as, in practice, one may want to allow correlation
between the random effects. Expression of the Fisher information matrix has been proposed in
this case for single response in Mentré et al. [5] and for multiple responses in Ogungbenro et
al. [33] but has not been yet implemented in PFIM 3.0.

Afterwards, an extension of PFIM is envisaged to implement the expression of the Fisher
information matrix for models including fixed effects for the influence of covariates on the
parameters [6], and thus, to compute the predicted power of the Wald test to detect a covariate
effect as well as the number of subjects needed to achieve a given power [18].

Moreover, Retout et al. [12] have recently used a modified version of PFIM for the
problem of design optimisation using cost functions. They focus on the relative feasibility of
the optimised designs, in terms of sampling times and of numbers of subjects. To do that the
Fedorov-Wynn algorithm has been extended to include cost functions to penalize less feasible
designs. An illustration of this extension of design optimisation to a joint population model of
infliximab and methotrexate pharmacokinetics administered in rheumatoid arthritis was
performed. The possibility of specifying user-defined cost-functions will be available in the
next version of PFIM.

The implementation of the Fisher information matrix for multiple response models
proposed in PFIM 3.0 is a relevant computing tool for the evaluation and comparison of
designs in the spreading development of multiple response PK studies. PFIM 3.0 is
distributed under the terms of the GNU GENERAL PUBLIC LICENSE (GNU GPL) Version
2, June 1991 and has been registered at the Agency for the Protection of Programs (APP) in
PFIM functions, including their extensive documentation and examples to use them, are freely available from the PFIM website: http://www.pfim.biostat.fr.

Conflict of Interest Statement

The authors have no conflicts of interest that are directly relevant to the content of this work.
References


**Legends for Figures:**

Fig. 1: Model file of PFIM 3.0 for the evaluation of the empirical design $\Xi_{emp}$ and optimisation of both optimal designs $\Xi_{opt_{iden}}$ and $\Xi_{opt_{diff}}$ for the joint PKPD modelling of warfarin.

Fig. 2: Input file of PFIM 3.0 in the context of the evaluation of the empirical design $\Xi_{emp}$ for the joint PKPD modelling of warfarin.

Fig. 3: Input file of PFIM 3.0 in the context of the optimisation of the optimal design $\Xi_{opt_{iden}}$ using the Fedorov-Wynn algorithm for the joint PKPD modelling of warfarin.

Fig. 4: R graph output of the concentration and effect profiles versus time for the mean parameter values used in the joint PKPD modelling of warfarin using PFIM 3.0. PK sampling times and PD sampling times for the optimal design $\Xi_{opt_{iden}}$ for each response are displayed using number values (i.e. 1 and 2 for samples from first and second elementary design, respectively).
Table 1. List of pharmacokinetic models included in the library of models of PFIM 3.0. The table includes all information needed in order to use the model function chosen: the name, the type of input, the number of compartments and the parameters used.

<table>
<thead>
<tr>
<th>Name</th>
<th>Input</th>
<th>Number of compartment</th>
<th>Parametrisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>bolus_1cpt_Vk</td>
<td>IV-bolus</td>
<td>1</td>
<td>V, k</td>
</tr>
<tr>
<td>bolus_1cpt_VCl</td>
<td>IV-bolus</td>
<td>1</td>
<td>V, Cl</td>
</tr>
<tr>
<td>infusion_1cpt_Vk</td>
<td>IV-infusion</td>
<td>1</td>
<td>V, k</td>
</tr>
<tr>
<td>infusion_1cpt_VCl</td>
<td>IV-infusion</td>
<td>1</td>
<td>V, Cl</td>
</tr>
<tr>
<td>oral1_1cpt_kaVk</td>
<td>1st order</td>
<td>1</td>
<td>ka, V, k</td>
</tr>
<tr>
<td>oral1_1cpt_kaVCl</td>
<td>1st order</td>
<td>1</td>
<td>ka, V, Cl</td>
</tr>
<tr>
<td>bolus_2cpt_Vkk12k21</td>
<td>IV-bolus</td>
<td>2</td>
<td>V, k, k12, k21</td>
</tr>
<tr>
<td>bolus_2cpt_ClV1QV2</td>
<td>IV-bolus</td>
<td>2</td>
<td>Cl, V1, Q, V2</td>
</tr>
<tr>
<td>infusion_2cpt_Vkk12k21</td>
<td>IV-infusion</td>
<td>2</td>
<td>V, k, k12, k21</td>
</tr>
<tr>
<td>infusion_2cpt_ClV1QV2</td>
<td>IV-infusion</td>
<td>2</td>
<td>Cl, V1, Q, V2</td>
</tr>
<tr>
<td>oral1_2cpt_kaVk12k21</td>
<td>1st order</td>
<td>2</td>
<td>ka, V, k, k12, k21</td>
</tr>
<tr>
<td>oral1_2cpt_kaClV1QV2</td>
<td>1st order</td>
<td>2</td>
<td>ka, Cl, V1, Q, V2</td>
</tr>
</tbody>
</table>
Table 2. Population parameter values estimated by the PKPD joint modelling of warfarin

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fixed effects ($\beta$)</th>
<th>Inter-subject variability ($\omega^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_a$ (h$^{-1}$)</td>
<td>1.60</td>
<td>0.701</td>
</tr>
<tr>
<td>$CL$ (L/h)</td>
<td>0.133</td>
<td>0.063</td>
</tr>
<tr>
<td>$V$ (L)</td>
<td>7.95</td>
<td>0.020</td>
</tr>
<tr>
<td>$R_{in}$ (%/h)</td>
<td>5.41</td>
<td>0.190</td>
</tr>
<tr>
<td>$IC_{50}$ (mg/L)</td>
<td>1.20</td>
<td>0.013</td>
</tr>
<tr>
<td>$k_{out}$ (h$^{-1}$)</td>
<td>0.056</td>
<td>0.016</td>
</tr>
<tr>
<td>$\sigma_{slopePK}$</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{interrPD}$ (%)</td>
<td>3.88</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 3. Optimal population designs for the joint model of warfarin concentration (PK) and the effect on prothrombin complex activity (PD) according to different constraints. \( \Xi_{\text{emp}} \) is the empirical design. \( \Xi_{\text{opt iden}} \) and \( \Xi_{\text{opt diff}} \) are both optimal designs. \( \Xi_{\text{opt diff simp}} \) is a design derived from \( \Xi_{\text{opt diff}} \) to be simpler. For each design, the number of elementary designs, the rounded number of subjects with the elementary design, the sampling times for each response, the total number of sampling times and the associated criterion value are reported.

<table>
<thead>
<tr>
<th>Number of elementary designs</th>
<th>Sampling times for each response (hours)</th>
<th>PK</th>
<th>PD</th>
<th>Rounded number of subjects</th>
<th>Total number of sampling times</th>
<th>Criterion value ( \Phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Xi_{\text{emp}} )</td>
<td>0.5, 1, 2, 3, 6, 9, 12, 24, 36, 48, 72, 96, 120</td>
<td>0, 24, 36, 48, 72, 92, 120, 144</td>
<td>32</td>
<td>672</td>
<td>984.20</td>
<td></td>
</tr>
<tr>
<td>( \Xi_{\text{opt iden}} )</td>
<td>0.5, 12, 24, 144</td>
<td>0.5, 12, 24, 144</td>
<td>22</td>
<td>256</td>
<td>580.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5, 24, 120, 144</td>
<td>0.5, 24, 120, 144</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi_{\text{opt diff}} )</td>
<td>0.5, 6, 9, 144</td>
<td>0, 12, 24, 120</td>
<td>3</td>
<td>256</td>
<td>607.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5, 6, 9, 144</td>
<td>0, 24, 96, 120</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Xi_{\text{opt diff simp}} )</td>
<td>0.5, 6, 9, 144</td>
<td>0, 24, 96, 120</td>
<td>32</td>
<td>256</td>
<td>607.32</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Relative standard errors (RSE) (%) computed by the population Fisher information matrix for designs $\Xi_{\text{emp}}$ and for the enlarged designs $\Xi_{\text{opt iden enl}}$ and $\Xi_{\text{opt diff simp enl}}$ derived from the two optimal designs by increasing the number of subjects in order to get a similar efficiency obtained with $\Xi_{\text{emp}}$. The total number of sampling times and the criterion value associated with each design are also reported.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{k_s}$</th>
<th>$\beta_{CL}$</th>
<th>$\beta_{\psi}$</th>
<th>$\beta_{K_s}$</th>
<th>$\omega^2_{k_s}$</th>
<th>$\omega^2_{CL}$</th>
<th>$\omega^2_{\psi}$</th>
<th>$\omega^2_{K_s}$</th>
<th>$\omega^2_{K_{IC}}$</th>
<th>$\sigma_{\text{slope PK}}$</th>
<th>$\sigma_{\text{interPD}}$</th>
<th>Total number of sampling times</th>
<th>Criterion value $\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi_{\text{emp}}$</td>
<td>15.7</td>
<td>4.6</td>
<td>2.9</td>
<td>8.0</td>
<td>3.3</td>
<td>2.9</td>
<td>28.0</td>
<td>26.3</td>
<td>33.0</td>
<td>26.2</td>
<td>68.3</td>
<td>40.8</td>
<td>3.9</td>
</tr>
<tr>
<td>$\Xi_{\text{opt iden enl}}$</td>
<td>12.5</td>
<td>3.7</td>
<td>3.1</td>
<td>6.1</td>
<td>3.4</td>
<td>2.3</td>
<td>22.4</td>
<td>21.1</td>
<td>45.1</td>
<td>20.2</td>
<td>98.0</td>
<td>34.9</td>
<td>8.2</td>
</tr>
<tr>
<td>$\Xi_{\text{opt diff simp enl}}$</td>
<td>12.7</td>
<td>3.8</td>
<td>2.8</td>
<td>6.3</td>
<td>3.2</td>
<td>2.6</td>
<td>22.9</td>
<td>21.8</td>
<td>38.9</td>
<td>20.8</td>
<td>83.6</td>
<td>41.7</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>672</td>
<td>984.2</td>
<td>440</td>
<td>997.2</td>
<td>416</td>
<td>987.5</td>
<td>984.2</td>
<td>997.2</td>
<td>987.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
formED <- function (t, y, p)
{
  ka <- p[1]
  cl <- p[2]
  V <- p[3]
  Rin <- p[4]
  C50 <- p[5]
  kout <- p[6]
  Imax <- 1

  yd1 <- ka * y[1]

  list(c(yd1, yd2, yd3), c(y[2] / V, y[3]))
}
# Name of the project
project="FVFM/Thermal_indirect_model: empirical design evaluation"

# Name of the file containing the RE or RD model
file.model="model.r"

# Name of the output file for the results
output="output.r"

# Run: Simulation (EVAL) or Optimisation (OPT)
run="EVAL"

# Number of responses
nr=2

# Name of the fixed effects parameters
parameters="c('ka','cl','V','rin','C50','hour')"

# Fixed effects parameters values
\textbf{beta}=c(1.6, 5.133, 7.95, 5.41, 1.2, 0.056)

# diagonal matrix of variance for the random effects
\textbf{omega}=diag(c(0.701, 0.063, 0.02, 0.019, 0.0329, 0.036))

# Standard deviation of the residual error, \(\text{sig.int}+\text{sig.slope}\)
\textbf{sig.int}=0
\textbf{sig.slope}=0.2
\textbf{sig.int.slope}=3.88
\textbf{sig.slope.slope}=0

# between-subject variance model (1) = additive (2) = exponential
\textbf{Trand}=2;

# List of the vectors of sampling times for each elementary design
\textbf{protA}=list(c(0.5, 1, 2, 3, 6, 9, 32, 24, 36, 48, 72, 96, 120))
\textbf{protB}=list(c(0.24, 36, 48, 72, 96, 120))

# Vector of initial proportions or numbers of subjects for each elementary design
subjects=c(32)

# subjects input: [1] for number of subjects (2) for proportions of subjects
subjects.input=1

# if proportions of subjects: give the total number of samples
\textbf{N}=1000

# MODE OPTION
# Model form: Differential equations (DE) or analytical form (AF)
modelform="DE"

# Initial time for which initial conditions are given
\textbf{time.init}=0

# Identical initial conditions in each elementary design (Yes=T, No=F)
condition.identical=T

# If 'Yes', enter once the expression of the initial values of the system in the initial time
# else, enter the vectors of the initial conditions for each elementary
# If initial values depend on parameters to be estimated, enter this parameter into the expression without any quotation marks
\textbf{init=(expression(120, Min, Max))}

# Error tolerance for solving differential equations
\textbf{EREL}=1e-08
\textbf{ATOL}=1e-08
\textbf{RMAX}=Inf # Default value
\textbf{NMAX}=200 # Default value

# END DIFFERENTIAL EQUATION OPTION

Figure
Fig. 3

FDIM 3.6
Project: PERMWarfarine_indirect_model : design optimisation
Date: Tue Mar 17 10:17:00 2009

********************************************************** INPUT SUMMARY **********************************************************
Initial optimisation design:
Sample times for response: A
Protocol subjects
1 c(i:0.5, 12, 24, 92) 32 c(120, 8, 10, Rin/kout)

Sample times for response: B
Protocol subjects
5 c(i:0.5, 12, 24, 92) 32 c(120, 8, 10, Rin/kout)

Initial Conditions at time 0:
160 0 Rin/kout

Variance error model response A: (0 + 0.2 * f)^2
Variance error model response B: (3.88 + 0.5 * f)^2
Between-subject variance model: Tand = 2
Total number of samples: 256

Associated criterion value: 475.2673

Sampling windows for the responses: A
Window 1: x = 0.05 2 6 9 12 24 36 48 72 96 120 144
Maximum number of points in one elementary protocol = 5
Minimum number of points in one elementary protocol = 5

Sampling windows for the responses: B
Window 1: x = 0.05 2 6 9 12 24 36 48 72 96 120 144
Maximum number of points in one elementary protocol = 4
Minimum number of points in one elementary protocol = 4

How evaluating the Fisher Information Matrix for the 495 protocols generated
********************************************************** OPTIMISED DESIGN **********************************************************
Optimised population design:
Sample times for response: A
Times from Subjects
1 c(i:0.5, 12, 24, 144) 0.6768466 21.95529 c(10, O, Rin/kout)
2 c(i:0.5, 24, 120, 144) 0.3231534 10.429 c(10, O, Rin/kout)

Sample times for response: B
Times from Subjects
1 c(i:0.5, 12, 24, 144) 0.6768466 21.95529 c(10, O, Rin/kout)
2 c(i:0.5, 24, 120, 144) 0.3231534 10.429 c(10, O, Rin/kout)

Associated optimised criterion: 580.1969

 *************************************** POPULATION FISHER INFORMATION matrix ***************************************

[1,] [1.5] [-2.12196 241 -2.820803 -2.640843] 0.000000e+00
[2,] [-2.2282027265] [-0.990334 -126.290323] 0.000000e+00
[3,] [-3.618086490] [-126.29032330.9306478] 0.000000e+00
[4,] [-0.060458577] [-126.29032330.9306478] 0.000000e+00
[5,] [0.21053275] [0.18303794 -0.2601373] 0.000000e+00
[6,] [-41.54738651] [126.530409 -236.075613] 0.000000e+00
[7,] [0.0000000] [0.000000 0.0000000] 3.927126e-03
[8,] [0.0000000] [0.000000 0.0000000] 1.586251e+00
[9,] [0.0000000] [0.000000 0.0000000] 1.586251e+00
[10,] [0.0000000] [0.000000 0.0000000] 1.586251e+00
[11,] [0.0000000] [0.000000 0.0000000] 6.128796e+01
[12,] [0.0000000] [0.000000 0.0000000] 1.022466e+02
[13,] [0.0000000] [0.000000 0.0000000] 1.404545e+02
[14,] [0.0000000] [0.000000 0.0000000] 5.987962e+02

 *************************************** EXPECTED STANDARD ERRORS ***************************************

<table>
<thead>
<tr>
<th>Beta</th>
<th>SE</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ka</td>
<td>1.660</td>
<td>0.263</td>
</tr>
<tr>
<td>cl</td>
<td>0.131</td>
<td>0.006</td>
</tr>
<tr>
<td>V</td>
<td>7.955</td>
<td>0.322</td>
</tr>
<tr>
<td>Rin</td>
<td>5.410</td>
<td>0.437</td>
</tr>
<tr>
<td>cmn</td>
<td>1.205</td>
<td>0.052</td>
</tr>
</tbody>
</table>

| kout | 0.356  | 0.001 | 3.031633 |

*************************************** Variance of Random Effects ***************************************

<table>
<thead>
<tr>
<th>Omega</th>
<th>SE</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>hm</td>
<td>0.728</td>
<td>0.428</td>
</tr>
<tr>
<td>cl</td>
<td>0.204</td>
<td>0.011</td>
</tr>
<tr>
<td>V</td>
<td>0.0256</td>
<td>0.011</td>
</tr>
<tr>
<td>Rin</td>
<td>0.150</td>
<td>0.055</td>
</tr>
<tr>
<td>cmn</td>
<td>0.0129</td>
<td>0.007</td>
</tr>
</tbody>
</table>

| kout | 0.0167 | 0.007 | 45.90337 |

*************************************** Variance of residual error ***************************************

<table>
<thead>
<tr>
<th>SIG</th>
<th>SE</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>sig.intercept</td>
<td>0.29</td>
<td>0.032</td>
</tr>
<tr>
<td>sig.slope</td>
<td>3.88</td>
<td>0.447</td>
</tr>
</tbody>
</table>

*************************************** DETERMINANT ***************************************

4.936610e+8

*************************************** CRITERION ***************************************

593.199

Time difference of 13.355 secs
sys.meff

1.62
Fig. 4.

Warfarin Concentration model
Initial Conditions = c(100, 0, Rin/kout)

PCA model
Initial Conditions = c(100, 0, Rin/kout)
Conflict of Interest Statement

The authors have no conflicts of interest that are directly relevant to the content of this work.