

A Vectorial Image Classification Method Based On Neighborhood Weighted Gaussian Mixture Model

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Abstract— The CT uroscan contains three to four time-spaced acquisitions of the same patient. Registration of these acquisitions forms a vectorial volume, which contains a more complete anatomical information. In order to outline the anatomical structures, multi-dimensional classification is necessary for analyzing this vectorial volume. Because of the partial volume effect (PVE), probability distributions are assigned to the different material types within this vectorial volume instead of a definite material distribution. Gaussian mixture model is often used in probability classification problems to model such distributions, but it relies only on the intensity distributions, which will lead a misclassification on the boundaries and inhomogeneous regions with noises. In order to solve this problem, a neighborhood weighted Gaussian mixture model is proposed in this paper. Expectation Maximization algorithm is used as optimization method. The experiments demonstrate that the proposed method can get a better classification result and less affected by the noise.

I. INTRODUCTION

THE CT uroscan is the classical preoperative examination for renal surgery. It consists of three to four time-spaced 3D acquisitions at several contrast medium diffusion stages, which give complementary information about the kidney anatomy. Since information from these acquisitions is of a complementary nature, it is useful for the surgeon to integrate this information within a unique spatial volume. The first step in this integration process is to bring the different acquisitions into spatial alignment which has been done through a local mutual information maximization registration technique [1].

After this registration process, we get a volume, in which each voxel contains a vector of n elements corresponding to the information of the CT uroscan acquisitions (n is equal to the number of acquisitions, three to four in our case). For analyzing this volume, a multi-dimensional classification should be performed.

Due to partial volume effect (PVE), the object boundary voxels' values are usually the combination of two materials. Getting the material probabilities instead of assigning a definite material to the boundary voxels will be more conformable to the reality. Statistical classifiers have the

ability to get the probability distributions. Gaussian probability density functions are widely used to model the distribution of values within a dataset [2-4]. If K is the number of classes, we assumed that each voxel is composed by K component densities mixed together with K mixing coefficients. Each class density follows an n -dimensional Gaussian distribution, where n is equal to the number of elements in each voxel.

Unfortunately, firstly these intensity classification methods rely only on the intensity distributions, which will lead to misclassification at the object boundaries. To understand misclassification, let us consider a situation where a dataset has three tissues A , B and C , with scalar values $f(a)$, $f(b)$ and $f(c)$, respectively, such that $f(a) < f(b) < f(c)$. Let us assume that the tissues A and C touch each other, chances are very high that the boundary between A and C is classified to B . In addition, the lack of information during classification will lead to sensitiveness to the noises in inhomogeneous regions.

In order to solve the misclassification problem caused by intensity-only statistical classification methods, we proposed a neighborhood weighted solution. For analyzing a dataset, the information of neighborhood is also very important and the classification of the current voxel should take the neighborhood information into account. Based on this idea, a neighborhood weighted Gaussian mixture model is proposed in this paper.

The rest of this paper is organized as follows. Section II reviews some relevant previous works. The proposed model is presented in detail in Section III. Experimental results are illustrated and discussed in Section IV. Finally, the conclusions are given in Section V.

II. RELATED PREVIOUS WORKS

Within the class of intensity-based classification methods, Gaussian mixture model was widely applied on MR image segmentation [2-4]. But all these methods were applied to a single image (or volume) where each element to be classified is a scalar. These Gaussian mixture based methods can be easily expanded to a multi-dimensional situation by applying a multi-dimensional Gaussian distribution instead of a scalar one. And the solutions are just a simple expanding solvent, as implemented in this paper.

However, these intensity distribution based methods cannot solve the problem caused by PVE at the boundaries. S.A. Lakare [5] proposed a partial volume compensated classification method to solve this problem. When detecting a partial volume boundary, the author takes a compensated value instead of the sampled value. This method takes the

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PVE into classification process, but the classification result is still a definite decision at the partial volume boundary.

In order to get the correct material distributions at the partial volume boundaries, we proposed to take the neighborhood information, which is an important content of a volume, into the classification process.

Lunstrom *et al.*[6] proposed the Partial Range Histogram (PRH) concept, which is a way to describe the amount of a tissue within a local region. This gives us the hint to use this concept as a neighborhood descriptor. Inspired by this neighborhood description form, we propose a neighborhood weighted Gaussian mixture classification method with the purpose of getting a more accurate classification result.

III. PROPOSED CLASSIFICATION METHOD

In this section, the proposed method is presented. First, the general multi-dimensional Gaussian mixture model is described and solved by Expectation-Maximization (EM) algorithm. Then, the proposed neighborhood weighted method is described in detail. Based on this proposed model, the implemented algorithm is finally given.

A. Multi-dimensional Gaussian mixture model

For a joint-volume with N voxels, each voxel is a n -dimensional vector. The voxel intensity vectors are denoted by $x_i (i=1, 2, \dots, N)$. Recall that the goal is to estimate the class probabilities on each voxel according to the intensity vectors. The probability distribution of the k th tissue class is denoted by $p_k(x | \Theta_k)$, which is governed by a set of parameters Θ_k . Given the parameters of all the classes, the probability distribution of each voxel can be described as a mixture of probability distributions as follows:

$$p(x | \Theta) = \sum_{k=1}^K \alpha_k p_k(x | \Theta_k) \quad (1)$$

where α_k denotes the mixture coefficients. The parameter set of this distribution is $\Theta = (\alpha_1, \dots, \alpha_K, \Theta_1, \dots, \Theta_K)$ with the constraint that $\sum_{k=1}^K \alpha_k = 1$.

Typically, $p_k(x | \Theta_k)$ is modeled by a Gaussian distribution with mean μ_k and covariance matrix Σ_k . That is:

$$p_k(x | \Theta_k) = p_k(x | \mu_k, \Sigma_k) = \frac{1}{\sqrt{\det(2\pi\Sigma_k)}} e^{-\frac{(x-\mu_k)^T \Sigma_k^{-1} (x-\mu_k)}{2}} \quad (2)$$

Maximum likelihood (ML) estimation is a common used method to find the probability distribution parameters. The log-likelihood expression for this density from the data X is given by:

$$\begin{aligned} \log(L(\Theta | X)) &= \log \prod_{i=1}^N p(x_i | \Theta) \\ &= \sum_{i=1}^N \log \left(\sum_{k=1}^K \alpha_k p_k(x_i | \Theta_k) \right) \end{aligned} \quad (3)$$

Finding the ML solution directly from Eq. (3) is difficult because it contains the log of the sum. The EM algorithm is a good way to solve this problem [7]. The iterative solution for finding the parameters at the $(t+1)$ th iteration step is as follows:

$$\alpha_k^{t+1} = \frac{1}{N} \sum_{i=1}^N p(k | x_i, \Theta^t) \quad (4)$$

$$\mu_k^{t+1} = \frac{\sum_{i=1}^N x_i \cdot p(k | x_i, \Theta^t)}{\sum_{i=1}^N p(k | x_i, \Theta^t)} \quad (5)$$

$$\Sigma_k^{t+1} = \frac{\sum_{i=1}^N p(k | x_i, \Theta^t) \cdot (x_i - \mu_k^{t+1})(x_i - \mu_k^{t+1})^T}{\sum_{i=1}^N p(k | x_i, \Theta^t)} \quad (6)$$

Taking the mixing parameters α_k as prior probabilities, the probability of each class can be computed using Bayes' rule:

$$\begin{aligned} p(k | x_i, \Theta^t) &= \frac{\alpha_k^t p_k(x_i | \Theta_k^t)}{p(x_i | \Theta^t)} \\ &= \frac{\alpha_k^t p_k(x_i | \Theta_k^t)}{\sum_{j=1}^K \alpha_j^t p_j(x_i | \Theta_j^t)} \end{aligned} \quad (7)$$

B. Modified model with neighborhood information

Usually the material is continuous, so that it is natural to have the idea that for each voxel, the probability of the k th class should be affected by the neighbors' k th class probabilities. According to this belief, Eq. (7) should be modified.

Due to the deducing process of EM algorithm and the neighborhood idea, this probability should obey these rules:

$$1) \sum_{k=1}^K p(k | x_i, \Theta^t) = 1,$$

2) Current voxel's k th class probability magnifies if the neighbors' k th class probabilities tend to 1; current voxel's k th class probability decreases if the neighbors' k th class probabilities tend to 0.

Based on these two rules, we designed the neighborhood weighted probability for the current voxel:

$$p(k | x_i, \Theta^t) = \frac{\alpha_k^t W_{ik}^t p_k(x_i | \Theta_k^t)}{\sum_{j=1}^K \alpha_j^t W_{ij}^t p_j(x_i | \Theta_j^t)} \quad (8)$$

where

$$W_{ik} = \frac{\sum_{n=1}^{|N_i|} p(k | x_{n_i}, \Theta^t)}{|N_i|} \quad (9)$$

N_i is a set of neighborhood of the i th voxel. $|N_i|$ denotes the number of voxels in the set N_i . x_{n_i} denotes the n th neighbor's intensity of the i th voxel.

C. Description of the algorithm

Based on the discussions above, the estimation process we implemented is summarized as follows:

Input: The vectorial volume $x_i (i = 1, 2, \dots, N)$, the number of classes K .

Step 1: Initialization of Θ^0 and $p(k | x_i, \Theta^0)$. Any classification method could be used, in our case we choose K -means.

Step 2: Using Eq. (9) to calculate the neighborhood weight for each voxel.

Step 3: Calculate the prior probability by Eq. (8).

Step 4: Compute the new parameter data according to Eqs. (4), (5) and (6).

Step 5: Repeat steps 2-4 until reaching the end condition.

For each element vector of the input volume, the aim is to find its class distributions. From the iteration process, we can see that this algorithm is not limited in applying on vectorial volume. According to the spatial dimension of the input series $x_i (i = 1, 2, \dots, N)$ with N elements, denoted by D , the shape of the vectorial image to be classified can be a line ($D=1$), an image ($D=2$) or a volume ($D=3$). The difference is that the shape of N_i in Eq. (9) should match the dimension of the input series. Here, we only take the nearest neighbor into account with: $D=1, |N_i|=2$; $D=2, |N_i|=8$; $D=3, |N_i|=26$.

IV. EXPERIMENTS AND DISCUSSIONS

Experiments were performed on both synthetic and real data.

A. Evaluation on synthetic data

In order to illustrate the effect of classification, we use images to test our algorithm instead of volumes. We create an image where each pixel is a three elements vector ($n=3$). Each channel of the vector forms an independent image. The three images can be seen in Fig. 2. Each channel image is composed by two homogeneous regions on which we add some Gaussian noise. The combination of these three channels leads to a vectorial image with six classes. According to the proposed algorithm described in Section 3.3, the input number of classes $K=6$.

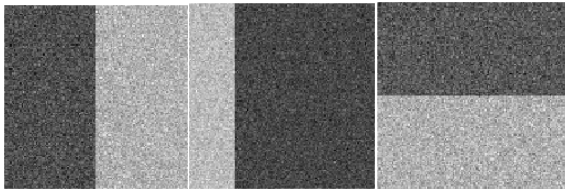


Fig. 2: Synthetic data. Each image is one channel of the vectorial image.

The classification on synthetic data is performed and the result is shown in Fig. 3. Each pixel of the result image is formed by this formula:

$$C(x_i) = \sum_{k=1}^K C_k p(k | x_i, \Theta) \quad (10)$$

where $C(x_i)$ is the color assigned to the i th pixel and C_k is the color we assigned to the k th class.

Fig. 3(a) is the classification result with the original Gaussian mixture model. We can notice that the final regions are not homogeneous as expected because of the noise. The reason is that the method relies only on the intensity distribution (histogram). The classification progress is a direct mapping from intensity to classes so that the noise cannot be removed. Fig. 3(b) is the result with our method. It is obvious to see that the regions are more homogeneous and the classification process is less affected by the noise.

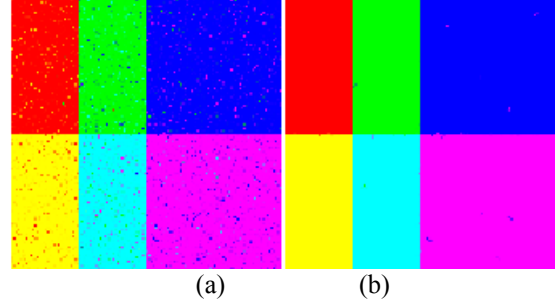


Fig. 3: Classification result of the synthetic data. (a): the Gaussian mixture method; (b): our method.

B. Application on real data

We performed the methods on real data obtained after the registration of three CT acquisitions. Fig. 4 shows one slice of the vectorial volume, which is composed by three channels: (a), acquisition before contrast medium injection; (b), immediately after injection; (c), ten minutes after injection.

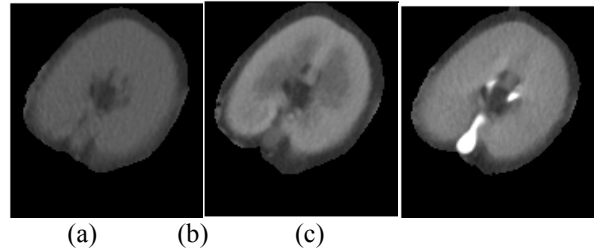


Fig. 4: One slice of the kidney volume after registration

With $K=4$, the classification result formed by Eq. (10) is shown in Fig. 5. It effectively demonstrates our conjecture. While taking the neighborhood information into account (Fig. 5(b)), the anatomical structures are better delineated into homogeneous regions: fat (red), renal cortex (green), renal medulla (blue) and collecting system (white).

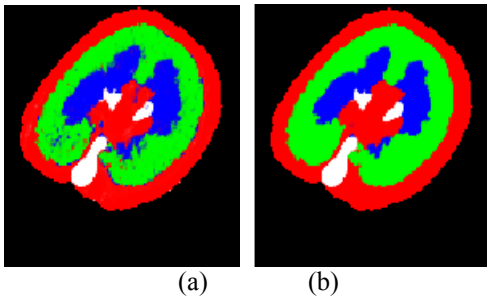


Fig. 5: Classification result of the real data. (a): the Gaussian mixture method; (b): our method.

C. Discussions

From the above results, we can see that the Gaussian mixture model based method has the ability to classify vectorial image with the aim of outline the anatomical structures. Because of the in-homogeneity of the acquisitions and the partial volume effects, the result of the intensity-only method has some misclassification area, especially the renal cortex and the renal medulla because of their close intensity range as seen in Fig. 5(a). In order to illustrate clearer this phenomenon, the first order derivate of the result probabilities along one cut line is shown in Fig. 6(a). We can clearly see that the probabilities within the regions are not homogeneous.

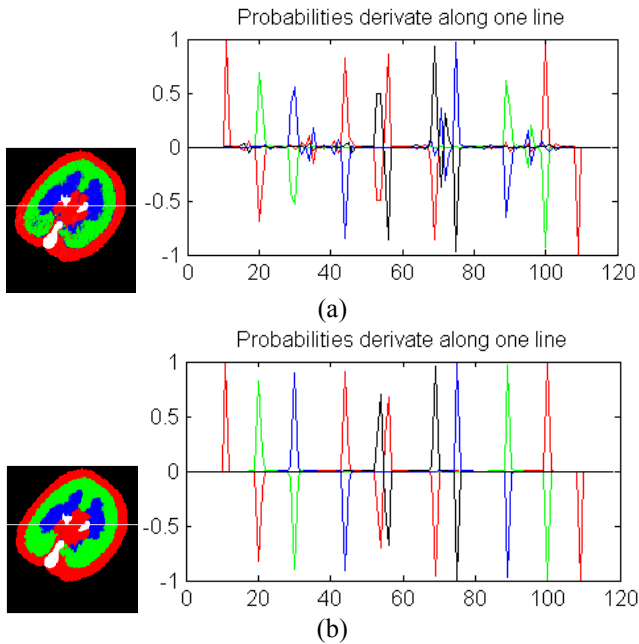


Fig. 6: Probabilities derivate along one cut line. (a): the Gaussian mixture method; (b): our method

When take the neighborhood information into the iteration process, the results are promoted significantly, as shown in Fig. 5(b). The proposed method considers the intensity and the position of one pixel simultaneously so that it can give a more reasonable classification decision. While comparing Fig. 6(a) and (b), we can see that besides avoiding PVE, it also has the effect of less sensitive to inhomogeneous region, while giving a more correct

classification decision.

V. CONCLUSIONS

A neighborhood weighted Gaussian mixture classification method is proposed and experimented in this paper. The model is that the voxels' intensity vectors follow the Gaussian mixture distribution and the classes distribution on each voxel is affected by its neighbors' class probability distributions so that a weight is used to describe this property. The mixture parameters are found by EM algorithm. Experiments on both synthetic and real data show that this Gaussian mixture model improvement is less affected by noise and gives better classification results.

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