Moment-based approaches in imaging part 2: invariance.

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Moment-based Approaches in Image.
Part 2: Invariance

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The several moment families have been reviewed in a first paper [1]. A classification was proposed in order to get a better understanding of their relations. More attention was given to orthogonal moments (in particular Legendre, Zernike, Tchebichef, Krawtchouk, Racah, dual Hahn). Important properties for computer vision applications were just sketched, among which invariance and robustness to noise. These properties may drive the choice of moments when addressing a specific problem. A short, thus non-exhaustive, review of the literature on these issues is proposed in this second paper.

Geometric Invariants

Theses invariants are perhaps the most critical for many applications, and certainly in medical imaging, when the objects are observed at different time instants and/or from different viewpoints. Translation, rotation and scaling of rigid objects are first of concern. If in some cases they are applied to 2-D images, i.e. tomographic slices or projections, indexed or not by time, the capability to extend moment invariants to 3-D is highly relevant. Geometric invariance has also a major interest in multimodal registration for diagnosis purpose or computer assisted therapy. Another key problem consists to deal with deformable objects. Elastic transformations are then required to estimate and compensate deformations that may be small or large. Moments may not be able to offer a solution to all these problems but they can provide some answers.

The pioneering impulse on the construction of moment invariants was given by Hu in the 1960s [2]. Based on the theories of invariant algebra, Hu derived seven moment invariants, computed from central moments through order three, that are independent to image scale, translation and rotation. Since then, many researchers have revisited his formulation and have proposed improvements and generalizations [3], [4]. Li [5] and Wong [6] reported moment invariants up to the orders nine and five, respectively. A general framework for constructing a large number of moment invariants was more recently proposed by Liu and Zhang [7]. Sadjadi and Hall [8] extended Hu's moment invariants to 3-D objects, based on two absolute moment invariants, with respect to translation, rotation and scale. Moment invariants dealing with a projective geometry has also been described by Suk and Flusser [9].

Rotation invariance has been extensively investigated in the past decades. Reddi [10] introduced the concept of rotational moments that are invariant under image rotation. Another set of rotation invariant moment functions, known as complex moments, was presented by Abu-Mostafa and Psaltis [11]. Teague [12] used the complex Zernike polynomials as the moment basis set to define Zernike moments. The separation of radial and angular dependence of Zernike polynomials makes them very attractive to extract the invariant features at random orientations. Moreover, the orthogonality property of Zernike polynomials avoids any redundancy between moments of different orders. Teh and Chin [13] presented a modification of Teague's Zernike moments based on pseudo-Zernike polynomials with similar properties. Generalized pseudo-Zernike moments have been reported in [14] for image description. Using the discrete Tchebichef polynomials and circular harmonic function as kernel, Mukundan [15] derived a set of radial Tchebichef moments invariant to image rotation. Other kinds of orthogonal moments that are invariant to image rotation include Fourier-Mellin moments [16], Chebyshev-Fourier moments [17], and radial harmonic Fourier [18].

The problem of scale and translation invariance of the orthogonal moments has been recently addressed. Chong et al. [19] presented a method to establish a set of scale invariants of pseudo-Zernike moments. Their approach was then used to construct both translation and scale invariants of Legendre moments [20]. Translation invariance of Zernike moments was also investigated by the same research group [21]. Scale and translation invariants of Tchebichef moments have been constructed by Zhu et al. [22]. It was shown that the methods developed in [19]-[22] have better performance than traditional approaches such as image normalization method and indirect method.

Some attention has been paid to the issue of completeness for moment invariants. Flusser et al. [23], [24] proposed a complete and independent set of rotation invariants by normalizing the complex moments. The construction of a complete set of similarity invariants (translation, rotation and scale) using linear combinations of complex moments has been studied by Ghorbel et al. [25]. It was shown that Flusser's set of invariants corresponds to a particular case of the similarity invariants presented in [25].

Convolution invariance and robustness to noise

Although more relevant in computer vision with variable scene illuminations than in medical imaging, uniform changes of image intensity function, simple to deal with, must be considered. Maitra [27] made Hu's invariants independent to contrast change. Wang and Healey [28] proposed illumination invariants that are particularly suitable for texture classification. However, the most important class of degradations is image blurring. Blurring may be caused by a number of factors, like out-of-focus cameras, small uncontrolled motions of the sensing device or the objects. If its impact remains low in medical imaging at large, it has nevertheless a high significance in optical imaging, with major applications in small animal imaging and biology [29]. Assuming that the imaging system is linear and space invariant, blurring can be described by a convolution of the original image with a point spread function (PSF). Since in most practical situations the PSF is unknown, finding a set of invariants that are not affected by blurring is a key problem in image analysis.

The pioneering work on this subject was performed by Flusser and Suk [30], [31] who derived moment invariants to convolution with any centrosymmetric PSF. Other sets of blur invariants were proposed for some particular kinds of PSF such as axisymmetric blur [32] and motion blur [33], [34].

If moment invariants based on the different moment types provide full invariance properties under noise-free condition, these properties may be affected in the presence of noise. Noise is of course inherent to any sensing devices and robustness to noise is highly desirable. However, multiple noise sources (i.e. sensing noise, reconstruction noise, etc.), with different features and non stationary properties, may corrupt the images. It was shown [13] that higher order moments are more sensitive to image noise than lower order moments, and that Zernike and pseudo-Zernike moments outperform the geometric moments and complex moments. Recent works [35], [36] have shown that the discrete orthogonal moments including Krawtchouk, Racah, and dual Hahn moments are very robust to noise when compared to the continuous orthogonal moments.

Table 1 summarizes the invariance properties and robustness to noise of different kinds of moments.

Combined invariants

Another important problem to address is when the images are subject to geometric transforms, blurring and noise degradations. In other words, multiple invariances have to be dealt with. A solution consists to sequentially address these problems. Another, a more elegant way, aims at simultaneously handling them. A few attempts have been devoted to such invariants, among which the most important contributions have been brought by Flusser and his group. Although initially examined in 2-D only (blur-shift or blur-rotation invariants), they were more recently extended to 3-D. For instance, Suk and Flusser [26] proposed a solution to deal simultaneously with affine transformation and blur (with centrosymmetric PSF) for pattern recognition, template matching and image registration. Flusser and Zivotá [37] suggested a set of combined moments which are invariant to both rotation and blurring. Based on the complex moments, Liu and Zhang [38] derived a subset of moment features that are not affected by image blurring and geometric transformation such as translation, scale and rotation. All these works however point out the problems related to the number of invariants to be selected, the choice of the region of interest size where moments are computed and the dependence with object features (i.e. symmetry).

Conclusion

This short survey of moment invariants points out that they bring interesting clues. Moments offer a sound theoretical framework for solving the generic problems encountered in many imaging applications. The diverse families of orthogonal moments provide the flexibility that may be required to face a particular target. They have however to satisfy the time computation constraints that are inherent to many applications. The next paper will show how moments can be approximated and their computation be accelerated.

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Type \ Property	Translation invariance	Rotation invariance	Scale invariance	Convolution Invariance	Robustness to noise
GM	Direct	Indirect	Direct	Direct	Low
CM	Direct	Direct	Direct	Direct	Low
RM	Indirect	Direct	Direct	Direct	Low
LM	Direct	Indirect	Direct	Indirect	Low
ZM	Direct	Direct	Direct	Indirect	High
PZM	Indirect	Direct	Direct	Indirect	High
CFM	Indirect	Direct	Direct	Indirect	High
GPZM	Indirect	Direct	Indirect	Indirect	High
TM	Direct	Indirect	Direct	Indirect	High
KM	Direct	Indirect	Indirect	Indirect	Very high
RAM	Direct	Indirect	Indirect	Indirect	Very high
DHM	Direct	Indirect	Indirect	Indirect	Very high

Table 1. A synthetic view on invariance and robustness to noise with [1]:
GM: Geometric Moments; CM: Complex moments; RM: Rotational Moments;
OM: Orthogonal Moments; LM: Legendre Moments; ZM: Zernike Moments;
PZM: Pseudo-Zernike Moments; CFM: Chebyshev-Fourier Moments;
GPZM: Generalized pseudo-Zernike Moments; TM: Tchebichef Moments;
KM: Krawtchouk Moments; RAM: Racah Moments; DHM: Dual Hahn Moments