Moment-based approaches in imaging part 3: computational considerations.

Huazhong Shu, Limin Luo, Jean-Louis Coatrieux

Moment-based Approaches in Image.

Part 3: Computational considerations

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Moment functions have been defined in [1] and important properties such as invariance and robustness to noise have been reviewed in the second paper [2]. Before addressing applications of moments, another feature has to be discussed, the computational load. The complexity of image analysis methods, in other words the number of operations they require to achieve a given task, iteratively or not, may lead to practical limitations when dealing with large data sets (2D or 3D image sequences) and time constraints. This issue is also of concern for moments in particular when high orders have to be computed. Special attention must therefore be paid to fast computation. The continuous-to-discrete transform may also affect the analytical properties we must preserve (i.e. invariance, orthogonality, etc.) by introducing numerical errors. The problem of accurate computation of moments should thus be addressed. These two aspects are examined in this third paper.

Accurate computation

Most of the moment functions are defined in continuous form. The double integration (refer to [1]) is usually approximated by a double summation. In order to increase the accuracy, Liao and Pawlak [3] proposed an improved version of the approximation formula for geometric and Legendre moments, further applied to Zernike moments [4]. More recently, Pawlak and his collaborators reported a novel scheme for high precision computation of Zernike moments in polar coordinate system [5]. Kotoulas and Andreadis [6] used a piecewise polynomial interpolation to get a more precise calculation of geometric moments. Jacob et al. [7] developed a method for the exact computation of geometric moments of a region bounded by a curve represented by smooth basis functions such as B-splines and other scaling functions. Sheynin and Tuzikov [8] proposed an algorithm for computing the geometric moments of a 2D object described by a spline curve boundary. In their method, the explicit formulae were derived.

It is worth noting that discrete moments such as Tchebichef, Krawtchouk, Racah and dual Hahn moments, do not suffer the problem of discrete approximation in their numerical implementation.

Fast algorithms

A significantly amount of computation is required to generate the moment values from images. Several options can be considered in order to accelerate the process by: (i) proposing new theoretical formulations; (ii) reducing the complexity; (iii) designing innovative implementations. The first two will be mainly addressed in this paper. Because the problem of fast computation of the geometric moments has been extensively investigated, one way for efficiently computing other kinds of moments such as Legendre and Zernike moments is to express them as a linear combination of geometric moments. Such a strategy was adopted by several research groups [6], [9]-[11]. It should be noted that most of the fast algorithms were focused on the use of the polynomial properties.

Fast computation of geometric moments

Many algorithms have been reported in the literature, either generic enough to deal with all types of images and object descriptions, or specific to well-defined situations (binary data or piecewise boundaries for instance). In an early work, Hatamian [12] used a causal spatial filter only requiring $O(N^2)$ additions for 2-D images with size $N\times N$. Zakaria et al. [13] proposed the so-called delta method for binary images. This method is suitable for images represented by y-lines, and was later improved by Dai et al. [14] and Li [15]. Note that Li's algorithm needs only $O(N)$ additions and multiplications for a convex object. Some fast algorithms make use of corner points of the object boundary [16]-[18]. Such approaches, limited to binary images, require $O(K)$ additions and multiplications where K denotes the number of corner points. By extending Jiang's algorithm, Li [19] suggested a fast algorithm for computing the 3-D image moments of polyhedra. Sheynin and Tuzikov [20] derived explicit formulae for this problem. Tuzikov et al. [21] presented a general and efficient approach for calculating surface moments of arbitrary-dimensional polytopes.

Another class of fast algorithms is based on the use of Green's theorem. Green's theorem evaluates the double integral over a region by a single integration along the boundary of the region. Li and Shen [22] described a fast method which requires $O(N)$ additions and multiplications. Their method, although efficient, relies on an approximation of Green's theorem. Using a discrete version of Green's theorem, Philips [23] suggested an exact calculation of image moments, less efficient however. Based on a new version of the discrete Green's theorem, Yang and Albregtsen [24] proposed a novel and exact algorithm for binary and gray-level images. Their method was then extended to 3-D moment computation [25]. Spiliotis and Mertzios [26] developed an efficient solution for binary images represented by blocks, later improved by Flusser [27] and generalized for gray-level images by Chung and Chen [28]. Local geometric moment computation has been dealt with by Martinez and Thomas [29]. Chung et al. [30] proposed an efficient computation of geometric moments based on the discrete cosine transform.

Most of the above mentioned algorithms were designed for cascade system (parallel implementation being addressed by Chen [31]). It should be pointed out that the solutions proposed by Chan et al. [32] and Liu et al. [33] only require additions for the fast computation of respectively 2-D and 3-D gray level image moments: they can also be implemented in a parallel mode.

Fast computation of orthogonal moments

As mentioned before, one way for computing the orthogonal moments is to express them as a linear combination of the geometric moments, and then to benefit of the previous fast algorithms. Another approach relies on the properties of orthogonal polynomials. Mukundan and Ramakrishnan [34] first used a Green's theorem, and then proposed a recursive scheme for computing the Legendre and Zernike polynomials. Shu et al. [35] derived an improved version for Legendre moments, later on extended to 3-D Legendre moments of polyhedra [36] in which the number of arithmetic operations depends only on I and J, where I and J represent respectively the boundary surface number and edge number of the polyhedra. Legendre moments of objects represented by y-lines have also been addressed by Zhou et al. [37]. Wang and Wang [38] described a recursive algorithm for the fast computation of the inverse Legendre moments.

Zernike moments have been extensively investigated as well in the past decades. Mukundan and Ramakrishnan [34] proposed a square to circular image transformation to simplify their computation. Belkasim et al. [39] used the radial and angular expansions of Zernike polynomials to speed up the algorithm. A recursive property of Zernike polynomials, where higher order polynomials are expressed as function of lower order ones, allowed Gu et al. [40] suggesting an iterative method. The reader can refer to a recent comparative analysis provided by Chong et al. [41]. Additional contributions have been reported since then. Wee et al. [42] suggested a hybrid algorithm to derive the subset of Zernike moments. Using the symmetry or anti-symmetry property of Zernike basis functions, Hwang and Kim [43] proposed a fast and accurate method. Chong et al. [44] developed a p-recursive method which uses a combination of lower order polynomials to derive higher order polynomials with same repetition q to improve the computation efficiency.

Recently, attention has also been paid to the fast computation of other orthogonal moments. Based on the symmetry property of Tchebichef polynomials, Mukundan [45] discussed the way to improve the computation of Tchebichef moments. Using Clenshaw's recurrence formula, Wang and Wang [46] proposed a recursive algorithm for computing the Tchebichef moments suitable for VLSI implementation. Kotoulas and Andreadis [47] presented a novel architecture suited for Tchebichef moments. Nakagaki and Mukundan [48] developed an algorithm for the fast computation of 4×4 discrete Tchebichef transform blocks.

Conclusion

An active research is devoted to improve both the accuracy and the efficiency of the computation of moments. The many situations to be handled according to the nature of the images, the object descriptions and the specific objectives that are pursued, make difficult to provide an exhaustive view and to precisely set the last achievements in terms of number of operations. As it has been shown, multiple options are explored for reducing their computational complexity and designing sound architectures. There is no doubt that faster algorithms are still needed to address more and more demanding applications in real-time environment, applications that will be surveyed in the next paper of this series.

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