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EEG SPIKE DETECTORS BASED ON DIFFERENT DECOMPOSITIONS: A COMPARATIVE STUDY

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I. INTRODUCTION

The EEG signal of epileptic patients exhibits some particular episodes where the most relevant are the seizures. Between seizures, the EEG is characterized by occasional epileptiform transients such as spikes and sharp waves. Because seizures do not occur frequently, the recording might require a long-term EEG monitoring. The detection of the interictal events is therefore of particular importance in the characterization of epilepsy and may have a high significance in terms of localization of epileptic foci. The interpretation of the underlying process (propagation routes, synchronism between brain regions) depends on the detection quality of the EEG segments where these transients are present. Gotman [1] gave an overview of the methods designed to recognize and quantify spikes, sharp-waves and spike-waves. Recent approaches have increased the detection performances by making use of the spatial and temporal context of the EEG [2] [3] [4]. Although an effort to automate the detection of epileptiform transients was undertaken, a complete solution has not been found yet. This is mainly due to the wide variety of shapes of these transient signals, their similarities to waves that are part of the background activity and to impulsive artifacts.

EEG signals observed over a period of time \([0, T]\) can be described, after sampling, by a random process \(X(k)\) whose form is:

\[
X(k) = F(k) + B(k) + \sum_{i=1}^{n_p} P_i(k-\theta_{p_i}) + \sum_{j=1}^{n_a} A_j(k-\theta_{A_j}) + B(k) ; \ k \in \{0, 1, ..., T\}
\]

This relation depicts relevant activities (elementary waves, background activity, noise, artifacts ...) which constitute the signal. \(F(k)\) may be considered as a piece-wise stationary signal present either casually or over the whole duration of the observation; for each \(i\), \(P_i\) represents a brief duration potential, with time occurrence \(\theta_{p_i}\), and corresponds to an abnormal neural discharge; the \(A_j\) terms may be related to artifacts occurring at times \(\theta_{A_j}\); finally, measurement noise which can be considered as stationary over the observation duration is described by \(B(k)\). Over the period of observation, the entities \(n_p\) and \(n_a\) represent respectively the number of temporal occurrences of brief useful events and artifact transient signals.

Cerebral background activity includes basic activities (Alpha, Beta, Delta, Gamma) as well as ictal stationary periods of time (recruitment phase during an epileptic seizure for example) and is modelled here by \(F(k)\). The distinction between the \(A_j\) and \(P_j\) components depends on the goals of the study: in our case, the epileptic events to detect are described by the \(P_j\) terms; accordingly, transitory waves associated to sleep, vertex sharp transients or K complexes, belong to the set of artifacts. Whatever the clinical objectives, the transient signals generated by eye movements or electrode shifts are represented by \(A_j\) terms.

From a signal processing point of view, the detection of spikes and sharp-waves can be seen as a classical detection problem where, at each time, the hypothesis "presence of spike" is confronted to its opposite. The difficulties encountered here come from the time-varying characteristics of both relevant signals (not perfectly known signals) and
noises (superposition of transient artifacts and locally stationary activities), and the unknown firing rate of the interictal events. To face the composite structure of the noises and the inherent non-stationary character of the spikes and artifacts, we proposed in [5] [6] a two-stages detection scheme based on time-scale representation of the observation X under the following hypothesis:

- \( F+B \) is approximately stationary on \( \{0, \ldots, T\} \) and is a zero mean gaussian signal with unknown covariance matrix,
- the frequency range of the artifacts \( A_j \) is statistically higher than the frequency band of the spikes \( P_i \),
- the transients being sparse events in the observation, first and second order statistics of \( F+B \) can be learned using \( X \),
- the shapes of \( A_j \) and \( P_i \) are random, only their duration is approximately known.

The proposed detection structure in [5] is based on two decision stages \( N_1 \) and \( N_2 \): the first one is a quadratic imposed structure aimed at the detection of \( P_i \) and the second one at the rejection of false alarms (of \( N_1 \)) due to \( A_j \). \( N_1 \) and \( N_2 \) use a wavelet filter bank determined heuristically.

In this chapter we compare the stage \( N_1 \) of the proposed detector to other quadratic detectors (with imposed structure or not) that make use of more or less information on transients and background activity. The problem is stated in the next section and the solution proposed in [5] is briefly presented. The new solutions for \( N_1 \) are introduced in section III. Experimental results and discussions are then reported.

II. PROBLEM STATEMENT

When dealing with the detection, over an observation time interval \([0, T]\), of an unknown number of transient signals whose supports are disjoined and arrival times are unknown, a classical sub-optimal solution, that we use here, is to consider the detection task like a sequence of elementary detection problems, each consisting in the detection of the presence of one transient on a short observation window covering the time support of the expected event. A serie of tests of the same type are conducted on a set of observation windows with equal length \( L \), the union of them being the whole observation time interval. More precisely:

- we define the following vectors:

\[
\begin{align*}
X(k) &= [X(k-L+1), \ldots, X(k)]^T, \quad k \in \{L-1, \ldots, T\} \\
F(k) + B(k) &= [F(k-L+1) + B(k-L+1), \ldots, F(k) + B(k)]^T \\
P(k) &= [P(k-L+1), \ldots, P(k)]^T \\
A(k) &= [P(k-L+1), \ldots, P(k)]^T
\end{align*}
\]

All these vectors are built, for each \( k \), in the same way, with \( L \) consecutive samples extracted respectively from the observation \( X \), the sum of instrumental noise \( B \) and background activity \( F \), the signal describing the spikes and the artifacts

- For each \( k \), we have to decide between two hypotheses:
\[ H_{0,k} : X(k) = F(k) + B(k) \quad \text{or} \quad X(k) = F(k) + B(k) + A(k) \]
\[ H_{1,k} : X(k) = F(k) + B(k) + P(k) \]

which are mutually exclusive because we neglect the probability of \( A(k) \neq 0 \) and \( P(k) \neq 0 \) on \( \{k-L+1, \ldots, k\} \). The hypothesis \( H_{0,k} \) can be decomposed in \( H_{0,k} = H_{0,k}' \cup H_{0,k}'' \) such that:

\[ H_{0,k}' : X(k) = F(k) + B(k) \]
\[ H_{0,k}'' : X(k) = F(k) + B(k) + A(k) \]

When assuming that the distribution of \( F+B \) is known (i.e. given or estimated with enough precision), the hypothesis \( H_{0,k} \) is simple while the hypotheses \( H_{0,k}' \) and \( H_{1,k} \) are composite because the shape of the transients and their arrival times are governed by a priori unknown distributions.

- We introduce here a serie of tests \( T_k \) between the hypothesis \( H_{0,k} \) and \( H_{1,k} \), and the corresponding serie of decision variables \( \delta_k \) which are set to 0 if \( H_{0,k} \) is true and to 1 otherwise.

- A "fusion" procedure on \( \delta_k \) is then used to ensure that 1) one transient signal leads at most to one detection, 2) the number of false alarms is not greater than 1 for a serie \( \{k_1, \ldots, k_2\} \) of decision instants such that \( k_2-k_1 \) is less than a given value (the minimal time duration of the spikes).

**DESCRIPTION OF THE TEST \( T_1 \)**

The heuristic detection algorithm proposed in [5] avoids the direct classical construction of the tests \( T_k \). It uses a wavelet filter bank, the outputs of which are processed by a two stage decision scheme. The first stage \( N_1 \) uses a serie of identical tests denoted \( T_1 \) between the hypothesis \( H_{0,k}' \) and \( H_{1,k} \) by ignoring the hypothesis \( H_{0,k}'' \). For each \( k \) we have \( \delta_k = 1 \) (\( H_{0,k}' \) true) if \( S_1(X(k)) > \lambda_1 \) otherwise \( \delta_k = 0 \) (\( H_{1,k} \) true). \( S_1 \) is a quadratic form and \( \lambda_1 \) is a threshold determined adaptively based on the empirical distribution of \( S_1(X(k)) \), \( k \) belonging to \( \{L, \ldots, T\} \). The second stage, \( N_2 \), is aimed at the elimination of the false alarms due to the presence, at the input of \( N_1 \), of artifacts \( A_j \). This stage has already been reported and will not be presented in this chapter. It has been shown experimentally (for detailed description see [5] and [6]) that most of the artifacts are removed through \( N_2 \).

The decision statistic, associated to the proposed detector is \( S_1(X(k)) = \| F X(k) \|^2 \) where \( F \) is a matrix such that
\[
F = \begin{bmatrix} F_1, F_2, \ldots, F_M \end{bmatrix}^T (t \text{ denotes the matrix transposition}) \]
has \( M \) rows \( F_i \) \( i = 1, \ldots, M \) which are FIR filters of a same length \( L \) and where \( \| Z \|^2 = Z^T Z \) for \( Z \) vector belonging to \( \mathbb{R}^L \). The detection test \( T_1 \) built in \( N_1 \) is of quadratic form (Figure 1) i.e. \( \| Y(k) \|^2 = F X(k) \|^2 > \lambda_1 \). As the transients \( P_i \) and \( A_j \) are brief and occur in the observation as "details", well localized in time-scale space, the matrix \( F \) was built based on a wavelet transform [7]. The \( F_i \) are then the sampled versions of the functions \( \psi_{a_i}(t) = \frac{1}{\sqrt{a_i}} \psi \left( \frac{t}{a_i} \right) \) \( i = 1, \ldots, M \), where \( a_i \) are scale values and \( \psi \) is a complex analyzing wavelet defined by:
\[
\psi(t) = C \cdot \left( 1 + \cos 2\pi f_0 t \right) e^{2\pi i k f_0 t} \| t \| \leq 1 / 2 f_0 , k \text{ integer } \mathbb{Z} \{-1,0,1\} \]

The parameter \( k \) sets up the number of oscillations of the complex part (admissibility conditions are verified for \( k \) different from \(-1,\, 0,\, 1\) ), \( f_0 \) is the normalized frequency and \( C \) is a normalization coefficient (\( \| \psi \| = 1 \)). The wavelet transform is then
exploited as a linear feature extraction procedure to obtain relevant time-scale atoms where the spikes $P_i$ are located. The $F_i$ were determined in [5] based on visual inspection of a large amount of interictal EEG data processed by wavelets (different patients and different interictal periods). The adaptive strategy presented in [5] allows a good control of the false alarm probability of $T_1$ both on simulated and real data. $S_1(X(k))$ can be expressed by the quadratic form $X(k)^tQX(k)$ where $Q = Q_o = F^tF$. The associated test is then noted $T_{1,o}$. Obviously, such construction of the decision test $T_{1,o}$ exploits heuristically the morphology of $P_i$ to determine the matrix $F$ and does not make a full use of the statistical properties of $F+B$ that can be achieved by learning on $X$. The main concern of this contribution is to propose alternative forms of $S_1$, in other words new matrices $Q$ by introducing more prior or learned information. For this purpose, a set of $N_E$ sampled spikes $\tilde{P}_r \in \mathbb{R}^t$, $r = 1, ..., N_E$ was selected manually from a real digitized EEG channel to build a dictionary of sampled spikes which is used to estimate the covariance matrix $C_p$ of $P_i$.

**VARIATIONS OF $S_1$**

All the proposed forms of $S_1$ are expressed by a quadratic function $S_1(X(k)) = X(k)^tQX(k)$, where $Q$ is set according to the selected criteria (each criteria gives to the test $S_1(X(k)) > \lambda_1$ a statistical signification). In the following, $F+B$ will be replaced by $B$ for simplification and the distribution assumed to be gaussian with the estimated covariance matrix $\hat{C}_B$.

**Detectors built without using the spike waveform**

- **Null hypothesis rejection test : $T_{1,N}$**

  The only required information on the signal is its duration. The test is based on the distance between the observation $X(k)$ and a null reference for the metric associated to $\hat{C}_B^{-1}$ where $\hat{C}_B$ is an estimate of the expectation of $X(k)X(k)^t$ (i.e. $E[X(k)X(k)^t]$) conditionally to the hypothesis $H_{0,k}$. The resulting quadratic form is obtained by setting $Q$ to $Q_N = \hat{C}_B^{-1}$.

- **Null hypothesis rejection test applied to $Y(k)$ : $T_{1,o}$**

  $Q$ is set to $F^t\hat{C}_Y^{-1}F$ where $\hat{C}_Y$ is an estimate of the expectation of $Y(k)Y(k)^t$ under the null hypothesis $H_{0,k}$. $T_{1,o}$ may be interpreted as $T_{1,N}$, with $Y$ instead of $X$.

**Detectors based on objective knowledge on $P_i$ (other than their time duration)**

In this section, the noise $B$ and spikes $P_i$ are supposed to be independent. With the help of the dictionary, the covariance matrix $C_p$ of the transients $P_i$ is estimated according to:

$$C_p = \frac{1}{N_E} \sum_{r=1}^{N_E} \tilde{P}_r\tilde{P}_r^t$$

- **Neyman Pearson detector : $T_{1,NP}$**
This detector is based on the likelihood ratio calculated assuming that the spikes are normally distributed with the estimated covariance matrix $\hat{C}_P$. The log-likelihood ratio is equal to $X^T(k)Q_{NP}X(k)$ (up to an additive constant) where $Q_{NP} = \hat{C}_B^{-1}(\hat{C}_B + \hat{C}_P)^{-1}$.

• **Deflection criteria detector** $T_{1,DC}$
The deflection associated to $S_1(X(k))$ is defined by:

$$D(S_1) = \frac{\left(\mathbb{E}_i[S_1] - \mathbb{E}_0[S_1]\right)^2}{\text{Var}_i[S_1]}$$

where $\mathbb{E}_i[S_1]$ and $\text{Var}_i[S_1]$ are the expectation and the variance of $S_1$ conditionally to the hypothesis $H_{i,k}$; $i = 0, 1$ respectively. The matrix $Q$ is obtained by maximizing the deflection $D(S_1)$. Consequently $Q$ is equal to $Q_{DC} = \hat{C}_B^{-1}\hat{C}_P\hat{C}_B^{-1}$.

• **Generalized likelihood ratio detector** $T_{1,GLR}$
The detector relies on the composite hypothesis $H_{1,k}$ which is defined by "$P_i$ is present and is localized in a given $d$-dimensional subspace, noted $E_p$, of $\mathbb{R}^L$". The generalized log-likelihood ratio between $H_{0,k}$ and $H_{1,k}$ leads to the statistic $S_1(X(k))$ where the matrix $Q$ is set to:

$$Q = Q_{GLR} = \hat{C}_B^{-1}G[G\hat{C}_B^{-1}G]^T[G\hat{C}_B^{-1}G]^T$$

where $G$ is a matrix of $d$ columns and $L$ rows, such that the column vectors span the space $E_p$. The hypothesis that the spikes are elements of $E_p$ is approximately verified if we suppose that the whole energy of the transient signals is localized in the subspace spanned by $d$ ($d < L$) eigenvectors corresponding to the $d$ largest eigenvalues of the covariance matrix $C_P$.

• **Stochastic extension of the matched filter**
The method described in [9] considers the signal to be detected as random and relies on a quadratic detection statistic $S_1(X(k))$ with imposed form. Moreover, $S_1(X(k)) = \|F'X(k)\|^2$ where $F' = [F_1', F_2', ..., F_{M'}']^t$ is a bank of $M'$ filters defined by the $M'$ eigenvectors associated to the $M'$ greatest eigenvalues of the matrix $\hat{C}_B^{-1}C_P$. For $M' = 1$, $F'X(k)$ is the linear form which maximizes a deflection criteria. When $M' > 1$ and assuming that the spikes distribution is gaussian, the authors argue that $S_1(X(k))$ is an approximation of the likelihood ratio. As it was conducted, the method supposes that the transients $P_i$ are stationary upon the time support of $X(k)$ and that their time locations are uniformly distributed over the time interval $[0, T]$, which lead to covariance matrices of stationary structures. The estimated covariance matrix is then a Toeplitz matrix noted $\hat{C}_{PS}$. According to our notations, $\hat{C}_{PS}$ is proportional to the square matrix of dimension $L$ whose elements are given by $\hat{C}_{PS(i,j)} = \frac{1}{N_0} \sum_{t=1}^{N_0} \sum_{i,j} \hat{P}_i(t)\hat{P}_j^T(k-(i-j))$. The above method was implemented and a modified version was proposed based on the following remarks: for the detection problem, we introduced the hypothesis $H_{1,k}$ which assumes implicitly that the whole energy of the spike is concentrated around the midpoint of $X(k)$. In other words, the spike is centred over the time support of $X(k)$. The spikes are then non stationary random signal and the estimate $\hat{C}_{PS}$ is replaced by $\hat{C}_P$. The original and modified statistics are associated to the tests $T_{1SMF}$ and $T'_{1SMF}$ respectively.
EXPERIMENTATION AND PERFORMANCE EVALUATION

The performance of the tests described above has been evaluated and compared on simulated data using Receiver Operating Curve (ROC) and modified ROC curves [8]. For each value of the decision threshold $\lambda_1$, the associated ROC curve point represent the probability $P_D$ for detecting a signal $P_i$, which is present in an observation window of length $L$, versus the false alarm probability $FAP$ defined by: $\Pr(S_1(X[k]) > \lambda_1 / H_{0,k})$, while the modified ROC curves represent the evolution of $P_D$ versus the false alarm rate $F\lambda\tau$ which corresponds, for large values of $\lambda_1$, to the mean value of the number of $\lambda_1$ up crossing of $S_1(X(k))$ per time unit.

A dictionary of $N_E = 100$ sampled spikes, extracted manually from an interictal EEG signal recorded using the standard protocol 10-20 and a sampling frequency equal to 200 Hz, was built and the covariance estimates were computed. According to the time duration of the spikes and to the frequency sampling, the observation vector dimension $L$ was set to 60 for all the statistics except for those based on wavelets ($T_{1,0}$ and $T'_{1,0}$) where the retained $a_i$'s led to $L = 30$. For the test $T_{1,GLR}$, the dimension of the space, generated by the covariance matrix of the spikes was set to $d = 4$. Hence the number of filters used for both original and modified stochastic extension of the matched filter is $M' = 4$. These choices were a compromise between the calculus complexity of $T_{1,k}$ at each step $k$ and the a priori information on signal and noise used (number of eigenvectors selected for $\hat{C}_P$ and $\hat{C}_B^{-1} \hat{C}_P$).

Before presenting experimental results, we must emphasize that the validation of detection methods like those mentioned above is a very difficult task. The main problems encountered are:

1) the definition of a set of EEG recordings to build a database with clinical significance. Signals are generally recorded over a long time period (several days) with a large number of electrodes, and they depend on the relative positions between cortical sources and electrodes. On the other hand, the shapes of epileptiform transients vary with patients and may change from an interictal period to another one.

2) The labelling of the signal components: it involves the detection of epileptiform transients by a visual inspection of the recordings and the validation by different physicians. Such procedure is subjective and expert dependent. It may discard some transients which could be detected in other circumstances or by another group of experts.

A complete evaluation of the above presented detectors on real data is far from being straightforward. Performances were then studied mainly by using simulation (artificially generated EEG) and preliminary results were obtained on real data.

RESULTS AND DISCUSSION

For performance analysis, artificial EEG signals were generated based on the following method: background activities were generated using autoregressive (AR) modelling of real EEG stationary periods without transients. 24 models were estimated based on classical criteria [10]. Abnormal EEG was obtained by superimposing periodically spikes,
randomly selected over the dictionary. An example of the simulated signal is presented in Figure 2. Based on the AR model's coefficient the matrices $C_B$ were calculated (one model corresponds to one matrix). Both false alarm probability and false alarm rate were measured on simulated background activities (204800 samples for each model). Two representative models were selected to illustrate the behavior of the detectors. Both ROC and modified ROC curves exhibit a similar hierarchy between the detection statistics. Over all the models, three groups can be distinguished. The best results are obtained for the Neyman Pearson test (the best), the deflection criteria test and the generalized likelihood test. The performances decreases when imposed quadratic structures (i.e. wavelet and stochastic matched filter) are used. The null hypothesis test leads to an intermediate level of performances. Among all the models, the modified stochastic matched filter extension detector as well as the modified wavelet transform detector perform better than their associated original versions (Figures 3a - 3b ; 4a - 4b ).

The tests on real data were conducted on the EEG signal from which the spike dictionary was built. $C_B$ was estimated on a spike free segment of the signal, while the estimates of $\hat{C}_P$ and $\hat{C}_{PS}$ are those used on synthetic EEG data. Figure 5 depicts the behavior of the detection statistics for an interictal EEG period of 10 seconds. The performance evaluations were made only through ROC curves. Figure 6 shows the behavior of the studied tests. The results point out that a new performance based hierarchy is exhibited. The Neyman-Pearson test and the null hypothesis rejection test are the worst. None of the tests perform better than the other ones for all false alarm probability values. However, two main points can be emphasized. Firstly, the introduction of the estimated covariance matrix $\hat{C}_y$ in the detector using wavelets (T',10) decreases significantly the performances especially for small values of FAP, while the use of $\hat{C}_P$ in place of $\hat{C}_{PS}$ improves the performance of the stochastic extension of the matched filter (T',1,SMF). Secondly, the test $T_{1,o}$ produces satisfactory results regarding the required a priori information (i.e. time-scale atoms where the spikes are located).

**CONCLUSION**

Different quadratic methods for detecting transients embedded in stationary background EEG activity were compared using simulated EEG. Some methods make use of objective information on the morphology of the waves to be detected (based on a dictionary of transient prototypes). All of the proposed structures, except the one proposed in [5], require an estimated covariance matrix of the background activity. Performance analysis in terms of both ROC curves and modified ROC curves, conducted on artificial interical EEG shows that a significant improvement of the quadratic tests performance is achieved when using the non stationary estimate of the spike covariance matrix. Original version of the stochastic extension of the matched filter can not be used as it is. The appropriate modification of the spike covariance estimate leads to better result. However, its performance remains lower than those obtained by means of the tests $T_{1,NP}$, $T_{1,GLR}$ and $T_{1,DC}$. The behavior of the wavelet tests $T_{1,o}$, $T'_{1,o}$ is similar to $T_{1,SMF}$. The first attempt made on real EEG data shows some modifications in the detectors hierarchy. However, we note that the best require the estimate of the covariance matrices of both background activity and spikes. Thus, the wavelet based detector $T_{1,o}$ is more attractive because such estimates are not required and its performance level is satisfactory. Furthermore, the procedure developed in [5] using the same wavelet decomposition scales to reject the artifacts detected due to the presence of $A_j$ and which leads
to good results, cannot be easily replaced by classical detectors because, in practice, the morphological informations on artifacts at our disposal are not precise enough. On real EEG data, the Neyman-Pearson detector and the null hypothesis rejection test exhibit the worst performances. Both tests assume that the background activity is stationary and normally distributed and the first (T_{1,NP}) supposes that the P_i are gaussian. Real signals may not comply with the above assumptions and it would be interesting then to investigate more extensively the domain of validity of such hypotheses and to study the robustness under non gaussianity.

The threshold (\(\lambda_1\)) determination was not discussed in this chapter. It can be evaluated adaptively, as presented in [5], to control the false alarm probability based on the T_{1,} samples (algorithm \(A_1\) in figure 1). Another approach is possible if the probability density law of T_{1,} under the null hypothesis, can be analytically determined as a function of the covariance matrix \(C_B\). In fact, using the analytical expression of the law, a threshold value is computed for a given false alarm probability (algorithm \(A'_1\) in figure 1). To control the false alarm rate, the algorithms \(A_1\) and \(A'_1\) require the joint density of T_{1,} and its variation between two consecutive samples.
REFERENCES


CAPTIONS

Figure 1 Block diagram of the quadratic detection structure.

Figure 2 Examples of 10 seconds simulated EEG signals : from top to bottom, 24 signals corresponding to 24 AR models, the spikes were added periodically (every 300 samples).

Figure 3-a ROC curves for the model 11. a) $T_{1,\text{NP}}$, b) $T_{1,\text{DC}}$, c) $T_{1,\text{GLR}}$, d) $T'_{1,\text{SMF}}$, e) $T_{1,\text{N}}$, f) $T'_{1,o}$, g) $T_{1,\text{SMF}}$, and h) $T_{1,o}$.

Figure 3-b Modified ROC curves for the model 11. a) $T_{1,\text{NP}}$, b) $T_{1,\text{DC}}$, c) $T_{1,\text{GLR}}$, d) $T_{1,\text{N}}$, e) $T'_{1,\text{SMF}}$, f) $T'_{1,o}$, g) $T_{1,\text{SMF}}$, and h) $T_{1,o}$.

Figure 4-a ROC curves for the model 14. a) $T_{1,\text{NP}}$, b) $T_{1,\text{GLR}}$, c) $T_{1,\text{N}}$, d) $T_{1,\text{DC}}$, e) $T'_{1,o}$, f) $T_{1,o}$, g) $T'_{1,\text{SMF}}$, and h) $T_{1,\text{SMF}}$.

Figure 4-b Modified ROC curves for the model 14. a) $T_{1,\text{NP}}$, b) $T_{1,\text{GLR}}$, c) $T_{1,\text{N}}$, d) $T_{1,\text{DC}}$, e) $T'_{1,o}$, f) $T_{1,o}$, g) $T'_{1,\text{SMF}}$, and h) $T_{1,\text{SMF}}$.

Figure 5 Detection statistics behavior on real EEG signal. A : Artifact, P : Spike. From top to bottom : raw EEG signal, $T_{1,\text{NP}}$, $T_{1,\text{DC}}$, $T_{1,o}$, $T'_{1,o}$, $T_{1,\text{N}}$, $T_{1,\text{SMF}}$, $T'_{1,\text{SMF}}$, and $T_{1,\text{GLR}}$

Figure 6 ROC curves obtained on a real EEG channel, a) $T_{1,\text{DC}}$, b) $T'_{1,\text{SMF}}$, c) $T_{1,o}$, d) $T_{1,\text{GLR}}$, e) $T_{1,\text{SMF}}$, f) $T_{1,o}$, g) $T_{1,\text{NP}}$ and h) $T_{1,\text{N}}$. 
- CB+F : covariance of B + F, estimated on X(0),..., X(T) (off line)
- CP : covariance of spikes estimated using the dictionary (off line)
- λ₁ : decision threshold, it may be calculated using \( \mathbf{C}_{B+F} \)
or estimated based on the empirical distribution of \( S_1(X(k)) \)
- A : threshold evaluation algorithm