

a cost function that is composed of a data fidelity term and a regularization term. For each projection angle ϑ , let $\mathcal{P}_\vartheta^{\text{geom}}(v) \in P(\vartheta)$ be the geometric cone-beam projection of the 3-D point v on the projection plane $P(\vartheta)$:

$$\mathcal{P}_\vartheta^{\text{geom}}(v) = \frac{a}{b + v_x \cos(\vartheta) + v_y \sin(\vartheta)} \begin{bmatrix} v_x \sin(\vartheta) - v_y \cos(\vartheta) \\ v_z \end{bmatrix}$$

where a (respectively, b) is the distance of the X-ray source to the detector (respectively, the volume center). We can now define the data cost of a 3-D skeleton V with respect to the projections at phase s

$$\mathbf{E}_s(V) = \frac{1}{N|V|} \sum_{v \in V} \sum_{n=1}^N D_{\vartheta_{n,s}}(\mathcal{P}_{\vartheta_{n,s}}^{\text{geom}}(v))$$

where $|V|$ denotes the cardinal of the set V , and $D_\vartheta : P(\vartheta) \rightarrow \mathbb{R}_+^*$ is a function calculating the distance to the extracted vessels, defined as follows. If v is a vessel endpoint, then $D_\vartheta(\mathcal{P}_\vartheta^{\text{geom}}(v))$ is equal to the square distance between $\mathcal{P}_\vartheta^{\text{geom}}(v)$ and the corresponding endpoint in the extracted vessel in $P(\vartheta)$. If v is not an endpoint, then we calculate $D_\vartheta(\mathcal{P}_\vartheta^{\text{geom}}(v)) = \Gamma(\vartheta)^{-1} \sum_{i=1}^{n_{\min}} \gamma_i(\vartheta) \|\mathcal{P}_\vartheta^{\text{geom}}(v) - c_i\|^2$, where $\Gamma(\vartheta) = \sum_{i=1}^{n_{\min}} \gamma_i(\vartheta)$, $c_1, \dots, c_{n_{\min}}$ are the n_{\min} closest points to $\mathcal{P}_\vartheta^{\text{geom}}(v)$ in the extracted vessel in $P(\vartheta)$, and the $\gamma_i(\vartheta)$ coefficients are weights that take into consideration local properties of $\mathcal{P}_\vartheta^{\text{geom}}(V)$, like the difference of directions between the projected vessel at point $\mathcal{P}_\vartheta^{\text{geom}}(v)$ and the segmented vessel at point c_i . The regularization term $\mathbf{F}(V)$ corresponds to a deformation energy and is equal to the normalized sum of the square distances between two neighboring points (in the sense of the topological structure defined earlier):

$$\mathbf{F}(V) = \frac{1}{\Upsilon(V)} \sum_{\ell \sim \ell'} \|v_\ell - v_{\ell'}\|^2$$

where $\Upsilon(V)$ denotes the number of cliques in V . Finally, for all $s \in \{2, \dots, S\}$, the deformation energy $\mathbf{D}_s(V)$ for the estimation of V_s is

$$\mathbf{D}_s(V) = \mathbf{E}_s(V) + \kappa \mathbf{F}(V)$$

where κ is a parameter that controls the elasticity of V . For $s = 2, \dots, S$, the estimation of V_s from V_{s-1} is performed as follows: at iteration (q) , points $v_\ell^{(q)}$ are displaced one by one in the gradient direction $\nabla_\ell \mathbf{E}_s(V^{(q)})$ (∇_ℓ denotes the gradient with respect to v_ℓ), with a time step $\delta_{q,\ell}$ equal to $N^{-1} \sum_{n=1}^N D_{\vartheta_{n,s}}(\mathcal{P}_{\vartheta_{n,s}}^{\text{geom}}(v_\ell^{(q)}))$, in order to slow down the motion as $V^{(q)}$ approaches the solution.

B. Motion Parameters Estimation

Once we have an estimation of each 3-D coronary tree model V_s , the next step is to estimate the registration functions that compensate the motion in the tomographic reconstruction: for each $s \in \{2, \dots, S\}$, we wish to build a function $\varphi_s : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that for each $\ell \in \{1, \dots, L\}$, $\varphi_s(v_\ell^s) \simeq v_\ell^1$. Let \mathcal{G} be a grid of Ω centered on the voxels and \mathcal{M} be a subgrid of \mathcal{G} . A B-spline parametric model is chosen to represent $\varphi_s = \varphi_{\alpha_s} = (\varphi_{\alpha_s}^X, \varphi_{\alpha_s}^Y, \varphi_{\alpha_s}^Z)$: for all $(c, C) \in \{x, y, z\} \times \{X, Y, Z\}$,

$$\varphi_{\alpha_s}^C(x, y, z) = c + \sum_{m=1}^{|\mathcal{M}|} \alpha_{X,s}^m b_m(x, y, z)$$

where $b_m(x, y, z) = b(x - x_m)b(y - y_m)b(z - z_m)$ is a cubic B-spline function centered on $(x_m, y_m, z_m) \in \mathcal{M}$. The estimation of

$\alpha_s = \{(\alpha_{X,s}^m, \alpha_{Y,s}^m, \alpha_{Z,s}^m)\}_{m=1}^{|\mathcal{M}|}$ is carried out by minimizing a least-squares cost function

$$\psi(\alpha_s) = \sum_{\ell=1}^L \|\varphi_{\alpha_s}(v_\ell^s) - v_\ell^1\|^2 + \mu \sum_{m \sim m'} \|\alpha_s^m - \alpha_s^{m'}\|^2 + \nu \|\alpha_s\|^2 \quad (3)$$

where the second sum is taken over the neighboring points of \mathcal{M} , and μ and ν are regularization parameters. By convention, $\alpha_1 = 0$.

III. TOMOGRAPHIC RECONSTRUCTION

In order to import the motion in an algebraic formula, we must compute a motion matrix $W(\alpha_s)$ that maps the volume vector at phase $s = 1$ to the volume vector at phase s , for each $s \in \{2, \dots, S\}$. The volume vectors $\mathbf{f}_1, \dots, \mathbf{f}_S$ must be redefined: let $f_1 : \Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}$ be the 3-D volume function at phase $s = 1$. We assume that f_1 can be written as the sum of B-spline functions $w_p(x, y, z) = w(x - x_p)w(y - y_p)w(z - z_p)$ centered on the voxel grid \mathcal{G} : $f_1(x, y, z) = \sum_{p=1}^{|\mathcal{G}|} u_p w_p(x, y, z)$. The transformation f_s of the image f_1 from the motion φ_{α_s} is a result of the composition¹ $f_s(x, y, z) = f_1(\varphi_{\alpha_s}(x, y, z)) = \sum_{p=1}^{|\mathcal{G}|} u_p w_p(\varphi_{\alpha_s}(x, y, z))$. Let us denote $\mathbf{f}_s = (f_s(x_1, y_1, z_1), \dots, f_s(x_{|\mathcal{G}|}, y_{|\mathcal{G}|}, z_{|\mathcal{G}|}))^T$, with $(x_j, y_j, z_j) \in \mathcal{G}$ for all $j \in \{1, \dots, |\mathcal{G}|\}$. Thus, we have the matrix formulation $\mathbf{f}_s = W(\alpha_s)\mathbf{u}$, where $\mathbf{u} = (u_1, \dots, u_{|\mathcal{G}|})^T$ and $(W(\alpha_s))_{j,p} = w_p(\varphi_{\alpha_s}(x_j, y_j, z_j))$. Our aim is now to estimate \mathbf{u} by solving the least-squares problem

$$\text{minimize} \sum_{s=1}^S \|\mathcal{P}_{\Theta_s} W(\alpha_s)\mathbf{u} - \mathbf{g}_s\|^2 + \rho Pr(\mathbf{u}) \quad (4)$$

such that $\forall p \in \{1, \dots, |\mathcal{G}|\}, u_p \geq 0$. Here, \mathcal{P}_{Θ_s} denotes the tomographic cone beam projector at angles $\Theta_s = \{\vartheta_{1,s}, \dots, \vartheta_{N,s}\}$ (please note that \mathcal{P}_{Θ_s} is different from $\mathcal{P}^{\text{geom}}$) and \mathbf{g}_s the corresponding observed projections. $Pr(\mathbf{u})$ is a vessel prior cost on \mathbf{u} defined as follows: for all $p = 1, \dots, |\mathcal{G}|$, let $\Delta_p = \min\{d^2((x_p, y_p, z_p), v_\ell^1), \ell = 1, \dots, L\}$ be the square distance of voxel p to V_1 . We define $Pr(\mathbf{u})$ as

$$Pr(\mathbf{u}) = \sum_{p=1}^{|\mathcal{G}|} \Delta_p |u_p|^\beta \quad (5)$$

where $\beta > 0$. This function penalizes high values for voxels that are located far from the centrelines. A similar prior has been used in [4].

IV. RESULTS

We simulated 3-D volumes of a coronary tree at 20 different cardiac phases, using 3-D centrelines V_1, \dots, V_{20} that had been extracted previously from a 3-D dynamic sequence acquired on a 64 slice general electric (GE) light-speed computed tomography (CT) coronary angiography [7]. This sequence included 20 volumes reconstructed at every 5% of the RR interval. The simulated dynamic volume is a sequence of binary functions $\mathbf{f}_s : \mathcal{G} \rightarrow \{0, 1\}$, such that $\mathbf{f}_s(v) = 1$ if v is located in a tube centered on the 3-D centerlines V_s and $\mathbf{f}_s(v) = 0$ in the opposite case. A Gaussian white noise with a standard deviation $\sigma = 0.05$ was added to the 2-D extracted centrelines coordinates, in order to simulate gating and 2-D centrelines extraction errors. The dimension of the voxel grid \mathcal{G} is $192 \times 192 \times 192$. \mathcal{G} is included in a cube Ω whose vertex coordinates are $(\pm 0.5, \pm 0.5, \pm 0.5)$. The number of cycles is 4

¹In theory, we should use a diffeomorphism ϕ_{α_s} that maps V_1 to V_s and perform the composition $f_1 \circ \phi_{\alpha_s}^{-1}$. Because our B-spline model is not "exactly" invertible, we used a function φ_{α_s} that approximately maps V_s to V_1 .

