Harmonic Magnification by Time Reversal based on a
Hammerstein Decomposition
Sébastien Ménigot, Jean-Marc Girault

To cite this version:
Sébastien Ménigot, Jean-Marc Girault. Harmonic Magnification by Time Reversal based on a Hammerstein Decomposition. IEEE International Ultrasonic Symposium 2016, Sep 2016, Tours, France. <hal-01371266>

HAL Id: hal-01371266
https://hal.archives-ouvertes.fr/hal-01371266
Submitted on 25 Sep 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution - ShareAlike 4.0 International License
Introduction
The medical ultrasound imaging systems have been improved by taking into account the nonlinear wave propagation. These improvements lead to increases the signal-to-noise ratio and the focusing in tissue harmonic imaging (THI). Since the backscattered non-linearities are a function of the transmitted signal, enhancing these nonlinearities means to design the best wave [1]. Time reversal process is well-known to optimize the SNR and free itself to phase aberration, by combining a waveform design in space and in time thanks a physical matched filter [2]. However; it is not well-adjusted for THI, because the propagation of the time reversed signal destroys the harmonics.

How can we optimize the SNR in tissue harmonic imaging?

As we want to guarantee a SNR optimization with a good focusing, the method has to include time reversal. The solution firstly consists in extracting the harmonic component at $2f_0$ by a Hammerstein filter. Then this time reversed harmonic component at $2f_0$ is frequency shifted to the fundamental component at $f_0$ and retropropagated in the medium.

Methods

1. Sending a first standard excitation $x_{\text{standard}}(n)$
2. Harmonic extraction based on a Hammerstein model where the nonlinear function is a frequency shifting by modulation (since it is a bijective function on $R$):
   \[ g(n) = \sum_{m=1}^{M} h_m(n) x_{\text{standard}}(n - m) \cdot C_p(n), \]  
   \[ n \text{ the discrete time, } M \text{ the memory of the Hammerstein model and } C_p(n) = \cos \left( 2\pi n \frac{f_0}{f_0} \right) \] with $f_0$ the sample frequency.
   The model can be solved by a pseudo-inversion:
   \[ h_m = (X^T X)^{-1} X^T y, \]  
   \[ h = [h_1(1), \ldots, h_1(M), h_2(1), \ldots, h_2(M)], \]  
   \[ y = [y(M+1), \ldots, y(N)]^T \] with $N$ the sample quantity, the matrix of input signals $X = [x_1 x_2]$ with
   \[ x = \begin{pmatrix} v_0(M) & v_0(M+1) & \cdots & v_0(N) \\ v_0(M+1) & v_0(M+2) & \cdots & v_0(N) \\ \vdots & \vdots & \ddots & \vdots \\ v_0(N) & 0 & \cdots & 0 \end{pmatrix}, \]  
   \[ v_0(n) = x(n) C_p(n). \] Finally, the second harmonic signal is: $y_{\text{NL}} = x_2^T [h_1(1), \ldots, h_2(M)]^T$.
3. Time reversing $y_{\text{NL}}$
4. Annihilation of the second harmonic effects of the second harmonics:
   \[ x_{\text{opt}}(n) = A \cdot y_{\text{NL}}(n) \cdot \cos \left( 2\pi f_0 \frac{n}{f_0} + \phi \right), \]  
   $A$ preserve the transmit power to $x_{\text{standard}}(n)$ and $\phi = -\arctan \left( \frac{\psi(y_{\text{NL}})}{\psi(x_2)} \right)$ with $\psi(y_{\text{NL}})$ the phase of the the second harmonic signal $y_{\text{NL}}$ and the $\psi(x_2)$ the phase of a cosinus at the frequency $f_0$. 

Simulation

\- 2D nonlinear wave propagation in a cavity [3]
\- 8-element probe centred at $f_0 = 4$ MHz
\- Initial Gaussian pulse $x_{\text{standard}}(n)$ centred at $f_0 = 22$ MHz and with a bandwidth of 50%