



## **”Examining Knowledge of Geometry : Response to Wulff and Delson”**

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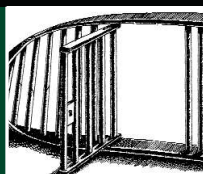
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## LETTERS

edited by Etta Kavanagh

### New Scientific Society in Nicaragua

IN MARCH OF THIS YEAR, *SCIENCE* PUBLISHED AN Editorial entitled "Fighting tropical diseases" (J. D. Sachs and P. J. Hotez, 17 Mar., p. 1521) discussing science-based approaches to address the UN Millennium Development Goals. Additionally, in the past few months, *Science* has reported on recent efforts to improve Africa's capacity in science, including the creation of science academies in various African countries. Because the motivation behind all these efforts is, in part, the urgent need to address extreme poverty, other low-income countries such as Nicaragua could benefit from similar initiatives.

Within Latin America, Nicaragua is a latecomer in promoting the use of science as a tool for economic development. Although scientific councils were created in most countries in the 1960s, in Nicaragua, it wasn't until 2002 that such a council (CONICYT) was first established. Even so, the lack of funding and working ties with the scientific community has rendered the council ineffective.

**"The organized engagement of scientists to make their voices heard could finally put science on Nicaragua's national agenda."**

—Huete-Pérez

Despite these difficulties, Nicaragua is currently experiencing a rapid period of scientific growth, particularly in the health sciences and biotechnology research. A recent report (1) by various members of the InterAmerican Network of Academies of Sciences (IANAS) praises the scientific achievements of Universidad Centroamericana and other Nicaraguan universities, but stresses that national planning and coordination are necessary for a stronger scientific enterprise.

In light of these recommendations and in an effort to build on this momentum, leading scientists have organized the Nicaraguan Society for Science as an essential step toward the creation of a Nicaraguan Academy of Sciences. In addition to providing independent advice to the government, such an academy would be able to help prepare a strategy to support science-based economic development.

For this effort to make headway, decision-makers will need to be persuaded of its value and be determined to allocate financial support. The organized engagement of scientists to make their voices heard could finally put science on Nicaragua's national agenda.

JORGE A. HUETE-PÉREZ

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1. H. Alper, M. Clegg, H. Ramkissoon, Report on the Inter American Network of Academies of Science (IANAS) visit to the Universidad Centroamericana, 16 to 18 Feb. 2006 (unpublished report).

### Examining Knowledge of Geometry

IN THEIR REPORT "CORE KNOWLEDGE OF GEOMETRY in an Amazonian indigene group" (20 Jan., p. 381), S. Dehaene *et al.* present evidence that an isolated Amazonian group, the Mundurukú, are able to understand geometric concepts. They state that geometry constitutes "a core set of intuitions present in all humans." I disagree with the basic concept of this investigation.

The central feature of Euclidean geometry is its demonstrative character and its logical structure, rather than graphical pictures of triangles, circles, etc. This logical system is built upon two pillars: (i) the concept of the "theoretical object," e.g., the abstract metaphysical idea of a circle, rather than a real constructed circle; and (ii) the deductive mathematical proof, based purely on axioms and postulates.

Other civilizations dealt with geometrical

figures in a more intuitive way, and their activities cannot be characterized as geometry in the Euclidean sense. Ancient civilizations other than the Greeks did not develop a demonstrative geometry. For example, the ancient Chinese never developed a theoretical geometry (1–3).

The topic being investigated by Dehaene *et al.* is simply pattern recognition. It is by no means surprising that the people tested recognized different geometric figures, since they can recognize, e.g., human faces and identify different species of tree by their silhouettes.

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2. G. F. Leibniz, *Novissima Sinica* (ed. 2, 1699), section 9.
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IN THEIR REPORT "CORE KNOWLEDGE OF GEOMETRY in an Amazonian indigene group" (20 Jan., p. 381), S. Dehaene *et al.* present research documenting the Brazilian Mundurukú Indians' ability to understand concepts of geometry and to orient themselves spatially. This team, as well as the scholars mentioned in C. Holden's article "Hunter-gatherers grasp geometry" (News of the Week, 20 Jan., p. 317), might be interested to learn that over 200 years ago, the great Brazilian naturalist Alexandre Rodrigues Ferreira also observed this innate ability of indigenes. In his memoirs of Amazonian zoology and botany [published collectively as the *Viagem Filosófica* (1)], he posed the question "what would a European, brought up like an Indian and ignorant of geometry, geog-



Amazonian Mundurukú villager reading a map to identify a hidden object.

raphy and hydrology, do if asked about a river's direction, branching and neighboring villages?" (p. 93, my loose translation). Anticipating Dehaene *et al.*, Ferreira conducted an experiment and asked this question of a Tapuio Indian, who by tying together several cords was able to create an approximate map of the local river and its tributaries, as well as point out the location of Indian villages. Further, Ferreira wrote that a Macuxí Indian he encountered not only drew an intelligible map of local river patterns and scaled hut outlines using a stick to trace lines in sand, but when presented with a pen and ink rendered the same idea on paper. Clearly, the naturalist understood native Brazilians not only to be reasoning individuals, but capable of understanding geometric and geographic concepts. Thus, on the basis of his own experiments, Ferreira would have agreed with Dehaene *et al.*'s conclusion that "geometrical knowledge arises in humans independently of instruction, experience with maps or measurement devices, or mastery of a sophisticated geometrical language."

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## Response

WULFF CONTRASTS PROCESSES OF DEDUCTIVE reasoning based on the axioms and postulates of Euclidean geometry with processes of visual pattern recognition, and he suggests that the latter processes underlie performance on our tests of geometrical categorization and map use. Our tasks were designed to assess geometrical concepts at a higher level of representation. Recognition of visual patterns is orientation-specific (1), yet both our tasks required that the Mundurukú abstract geometrical relations from figures that varied in orientation. The Mundurukú's globally high performance, particularly in the map task which requires a transformation from two to three dimensions, cannot plausibly be attributed to low-level processes of visual pattern recognition and implies extraction of genuine geometrical invariants.

Our tasks also do not depend on processes of deductive reasoning. Although geometry now appears as a beautiful logical construction, logic and deduction are neither necessary nor sufficient to account for core human geometrical concepts and intuitions. The central intuitions of Euclidean geometry cannot be deduced from simpler axioms, as the history of mathematics and physics attests (2): Absent the problematic and unprovable parallel postulate, Euclid's axioms and postulates support an infinite family of geometries at odds

with human intuition. Geometrical intuitions nevertheless come naturally to the human mind and continue to guide commonsense reasoning about space, even in scientists who have come to believe, by deduction and experiment, that the classical three-dimensional view of space fails to capture the structure of the universe (3).

We thank Delson for drawing our attention to Alexandre Rodrigues Ferreira's report. His informal observations on map-making appear to antedate ours by two centuries. Insofar as his observations were found to be general, they would confirm that the capacity to understand maps predates the most serious intrusions of Western culture. Caution is required in evaluating such ancient reports, but both Ferreira's report and our research suggest that all human cultures share an approximate arithmetic and intuitive geometry, which are highly stable over variations in education, language, and intercultural contact. On this point, we distance ourselves from claims of a radical "incommensurability" of cognitive functions in other cultures such as the Pirahã (4, 5), which are sometimes lumped together with our own views.

Research on core knowledge of geometry is at an early stage. How do geometrical concepts emerge in children? Are these concepts unique to humans or shared by other animals? What accounts for the distinctive profile of geometrical abilities shown both by indigenous tribes and by urban Americans? Converging studies across species, ages, and cultures, using methods of psychology and neuroscience, can begin to address these questions.

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## Ecological Revitalization of Chinese Villages

I READ R. STONE'S ARTICLE "VILLAGERS DRAFTED into China's model of 'sustainability'" (News of the Week, 7 Apr., p. 36) with great interest. In my 16 years investigating long-term ecological

changes in rural China, I have witnessed numerous governmental programs aiming to link improved rural livelihoods with ecological revitalization of village landscapes (1–3). Energy self-sufficiency and improved land management have been central to all of these programs, which have combined national and provincial policy and organizational efforts with demonstration villages, townships, counties, and even provinces. Although often only modestly successful, China's efforts to improve rural environments are an absolute triumph when compared with those of most rural developing countries and compare well with those of many developed countries during their own industrial transition.

The convergence of national and international environmental agendas on a single rural village will inevitably cause conflict. And Huangbaiyu will probably end up as have other village experiments: Temporary improvements will ultimately be dwarfed by the needs of local people to adapt to the developments going on around them. Although local demonstration projects might therefore be avoided, these can help to highlight and fix problems before more extensive programs are implemented. Regardless, the failings of a single village experiment should be presented as a minor part of the main story. China and the Chinese continue to make major efforts, by many means, to make the best of what is arguably the greatest environmental challenge any human population has ever faced (4).

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4. V. Smil, *China's Environmental Crisis, an Inquiry into the Limits of National Development* (M.E. Sharpe, New York, 1993).

## Stereotype Threat: A Clarification

THE REVIEW BY D. LEWIS (24 JUNE 2005, P. 1871) of the book *Gender Differences in Mathematics (1)* inadvertently perpetuates misinformation that has appeared elsewhere (2–4) about a key finding in our study of stereotype threat on an Advanced Placement calculus test (5, 6).

The study investigated whether asking women to record their gender immediately before taking the test elicited stereotype threat and thereby adversely affected their test performance. A quote in the review from a chapter in the book (4) cites an erroneous statistic from our initial technical

## Letters to the Editor

Letters (~300 words) discuss material published in *Science* in the previous 6 months or issues of general interest. They can be submitted through the Web ([www.submit2science.org](http://www.submit2science.org)) or by regular mail (1200 New York Ave., NW, Washington, DC 20005, USA). Letters are not acknowledged upon receipt, nor are authors generally consulted before publication. Whether published in full or in part, letters are subject to editing for clarity and space.

report (5), corrected in our subsequent article (6), that women asked their gender before the test scored significantly lower than those asked afterward. In fact, the mean difference between the two groups of women was not statistically significant. Another quote from the same chapter repeats an estimate (made by a referee in a privileged review of a draft of our article) that “as many as 2837 additional women per year” (about 47,000 took the test at the time of the study) would pass the test, earning advanced credit for calculus, if they were asked about their gender after the test rather than before it. This estimate is unreliable. It is based on rough calculations and extrapolations from the atypical sample of test takers in the study.

In short, this study of actual test taking, in common with our replication that examined community college students being given place-

ment tests for algebra and arithmetic (6), found no evidence of the deleterious effects of stereotype threat on the performance of women on quantitative tests that have been observed in laboratory experiments (4).

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\*Any opinions expressed in this letter are those of the author and not necessarily of Educational Testing Service.

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5. L. J. Stricker, *Inquiring About Examinees' Ethnicity and Sex: Effects in AP Calculus AB Examination Performance* (College Board Report No. 98-1, ETS RR No. 98-5, College Entrance Examination Board, New York, 1998).
6. L. J. Stricker, W. C. Ward, *J. Applied Soc. Psychol.* **34**, 665 (2004).

### Response

MY USE OF A QUOTATION INCLUDING “SIGNIFICANTLY” without qualification or explanation was indeed ambiguous and open to misinterpretation. Neither gender alone nor the interac-

tion of gender and the experimental–control (E-C) conditions was found to have a sufficiently large effect on AP grades to meet the standard of practical significance used by Stricker and Ward (1). However, the difference in the mean grades for the experimental and control groups of girls does meet a widely accepted standard of statistical significance, whether the means (2.62 and 2.41) reported in the original report (2) or those (2.61 and 2.41) reported in (1) are used.

Assuming normal means, the mean difference in boys' and girls' grades within the experimental group has the 95% confidence interval (3)  $0.16 \pm 0.18$ , while the mean difference within the control group is  $0.51 \pm 0.17$ . The mean difference for the experimental and control groups of girls is  $0.20 \pm 0.18$ ; that for boys is  $-0.15 \pm 0.16$ . (If the stereotype threat/lift hypothesis is correct, the experimental conditions should improve girls' performance and degrade boys'.) By way of comparison, note that in the introduction to their paper, Stricker and Ward describe the mean difference of 0.31 for boys and girls taking the exam in 1995 as substantial [(1), p. 669].

As regards the AP exam, practical significance seems to be the appropriate measure—the costs and possible unintended consequences of attempting to neutralize a very small

effect don't seem justified. However, given the inherent limitations of laboratory tests of stereotype threat—the stress and motivation associated with the AP, SAT, and GRE exams presumably aren't easily reproduced in the lab—and the obvious ethical and practical restrictions on experimentation with the actual exams, small but statistically significant differences resulting from apparently subtle changes in the exams may merit consideration as experimental results and as reality checks. If something as seemingly innocuous as filling in the appropriate gender bubble can have an effect on mathematical performance comparable in size to the effect of gender itself, we should think carefully about just how big the gender gap in mathematics really is, and what the effects of the popularization of that gap may be.

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#### References and Notes

1. L. J. Stricker, W. C. Ward, *J. Appl. Soc. Psychol.* **34** (no. 4), 665 (2004).
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3. Given two samples of size  $n_1$  and mean  $\mu_j$ ,  $j = 1, 2$ , with common standard deviation  $SD$ , the 95% probability confidence interval of the mean difference (assuming normal means) is approximately

$$\mu_1 - \mu_2 \pm 1.96 SD \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

The means are considered different if the interval does not contain the origin. See, e.g., (4).

4. "Multiple Comparisons: Comparison of Normal Means," in *Encyclopedia of Statistical Sciences* (Wiley, New York, 2005) (online version).

### CORRECTIONS AND CLARIFICATIONS

**Policy Forum:** "Incidental findings in brain imaging research" by J. Illes *et al.* (10 Feb., p. 783). On page 784, the DOI is incorrect; it should be 10.1126/science.1124665. The DOI is correct in the online version.

**Reports:** "Freezing as a path to build complex composites" by S. Deville *et al.* (27 Jan., p. 515). The symbols for author attributes were incorrect. S. Deville is the author to whom correspondence should be addressed (e-mail: sdeville@lbl.gov), and the present address for R. K. Nalla is Intel Corporation, 5000 West Chandler Boulevard, Chandler, AZ 85226, USA.

### TECHNICAL COMMENT ABSTRACTS

#### Comment on "HST2 Mediates *SIR2*-Independent Life-Span Extension by Calorie Restriction"

Matt Kaeberlein, Kristan K. Steffen, Di Hu, Nick Dang, Emily O. Kerr, Mitsuhiro Tsuchiya, Stanley Fields, Brian K. Kennedy

Calorie restriction (CR) increases life span in yeast independently of Sir2. Lamming *et al.* (Reports, 16 September 2005, p. 1861) recently proposed that Sir2-independent life-span extension by CR is mediated by the Sir2 paralogs Hst1 and Hst2. Contradictory to this, we find that CR greatly increases life span in cells lacking Sir2, Hst1, and Hst2, which suggests that CR is not mediated by Sir2, Hst2, or Hst1.

Full text at [www.sciencemag.org/cgi/content/full/312/5778/1312b](http://www.sciencemag.org/cgi/content/full/312/5778/1312b)

#### Response to Comment on "HST2 Mediates *SIR2*-Independent Life-Span Extension by Calorie Restriction"

Dudley W. Lamming, Magda Latorre-Esteves, Oliver Medvedik, Stacy N. Wong, Felicia A. Tsang, Chen Wang, Su-Ju Lin, David A. Sinclair

Our two labs and others have shown that *SIR2* controls the life span of diverse species, including *Saccharomyces cerevisiae* and *Drosophila melanogaster*, and that deleting *SIR2* blocks life-span extension by calorie restriction. The methods of Kaeberlein *et al.* allow yeast to bypass the requirement for *SIR2* and its homologs, which brings into question their suitability for modeling the physiology of more complex organisms.

Full text at [www.sciencemag.org/cgi/content/full/312/5778/1312c](http://www.sciencemag.org/cgi/content/full/312/5778/1312c)