Supplementary material to "Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures"

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I. Method

1. Participants

This work is part of a larger project on the nature of quantification and functional categories developed by the Unité Mixte de Recherche 7023 of the CNRS. It is based on psychological experiments and linguistics studies in the Mundurucu territory (Para) under the supervision of PP, in accordance with Consehlo de Desenvolvimento Cientifico et Tecnologicico and the Fundacão do Indio (Funaï; Processo 2857/04). The present data come from 35 experimental runs on the 1-10 task and 16 runs on the 1-100 task, collected during two field trips by P.P. in 2006 and 2007. A total of 33 Mundurucu participants were tested (2 were tested twice on the 1-10 task, on two different visits). Most of them came from an isolated area of Mundurucu Territory (up to 150 kilometers upstream of the Cururu mission). Eleven participants were children (age<18, range 7-17, average 11.9) and 22 were adults (range 18-78, average 39.7). All were native Mundurucu speakers. Only five were partially bilingual in Portuguese, although most could recite the first few Portuguese number words. Seven were adults who had received no schooling, others had received only a few years of education, with considerable variations in its intensity, duration and content. The first year of schooling typically focuses on basic alphabetization, the second year on basic writing, and the third year on the introduction of basic arithmetical concepts (addition, subtraction), this situation being subject to considerable variations mainly due to geographical location. For present purposes, we therefore separated participants who had been exposed to some schooling into those with little education (years 1-2; 15 participants) and those with more education (years 3 or more; 10 participants).
The control subjects were 16 American adults from the Boston area (age range 19-58, mean=32.8; 6 males). Although control subjects differed in their knowledge of Spanish, they were all familiar with Spanish number words and most of them could count up to 10 in Spanish.

2. Stimuli

Throughout the experiment with small numbers, a 25-cm horizontal segment was constantly present on screen, labelled with two reference sets. During the 1-10 runs, there was always one big dot at left and 10 smaller dots at right (this direction was arbitrarily chosen; whether the horizontal, left-to-right direction plays a privileged role in number-space mappings in the Mundurucu should be explored in future research). The target stimuli consisted of two practice sets of dots with numerosities 1 and 10 (later discarded); 20 sets with numerosities 1-10 (each presented twice); 10 sequences of tones 1-10; 8 Mundurucu numerals corresponding to 1, 2, 3, 4, 5, 7, 9 and 10 (see figure 1); and 10 Portuguese numerals 1-10. In a distinct block, the references presented at each end of the segment were 10 dots at left and 100 dots at right, and the target stimuli were 10 sets of dots ranging from 10 to 100 dots. Within each block, stimulus order was randomized. To prevent non-numerical confounds, the sets of dots were controlled according to a customary procedure (*1-3*): intensive and extensive parameters were varied separately for the reference sets and the target items, so that the reference stimuli were matched on total intensity and occupied area, but varied in item size and inter-item spacing, while the converse was true for target stimuli. The sequences of tones comprised 44-

millisecond beeps at 440 Hz, separated by a regular blank interval of 100 ms. The Mundurucu and Portuguese words were read aloud by a Mundurucu speaker, trained by P.P. to act as an experimenter on this task.

3. Procedure

During the two training trials, participants were successively shown the two reference quantities and were shown that these quantities belonged to the line endings. They were told that that the line represented a path from one quantity to the other, and that, for other quantities, they could point to any point on the line segment, anywhere between the left and right endpoints. They were asked to attend to quantity, and to point to the location where they thought it belonged. The approximate formulation was: “Here is a path. This path goes from “one” (pũg ma) to many (xep xep põgbi). Please place on the path the quantity x”. The response mode varied. American participants used the computer mouse to click to the appropriate screen location. They could correct their responses as needed, then pressed a key to move to the next trial. Mundurucu participants were asked the simple question “where now?” and asked to point to the screen using the eraser end of a pencil. They held up the pen until the experimenter (P.P., assisted in the first mission by his spouse Ana Arnor) had placed the computer cursor to the appropriate location (see figure S1). The experimenter could modify the placement of the cursor by clicking again, until he was satisfied of the correct placement, then he had to press a key to move to the next trial.

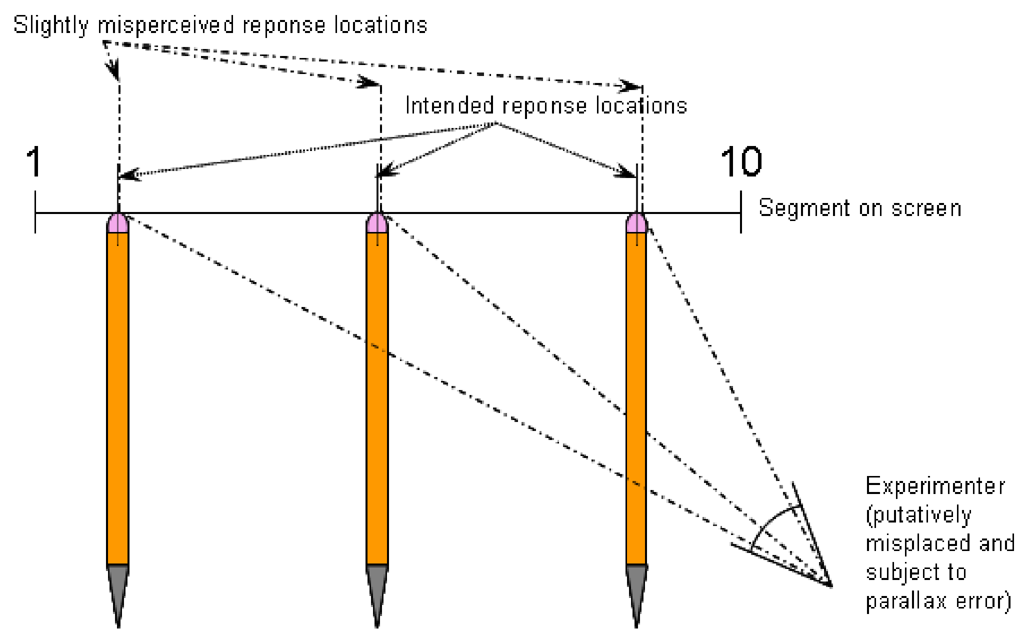
Figure S1. Response mode in Mundurucu participants (photo from experiment ran in Aldeia Muiussu (first mission, 2006) ; copyright Pierre Pica/UMR 7023).

II. Methodological concerns

1. Different response modes for Mundurucu and American participants

American participants responded directly with the computer mouse, while Mundurucu participants pointed to the screen and had their responses entered by the experimenter. One may ask whether this procedure might have introduced systematic parallax errors and/or a differential bias across the two groups, but such bias is unlikely for several reasons. First, all Mundurucu participants were tested individually by an experimenter seated behind the participant or slightly to the right. Thus, any parallax error should have shifted responses to the right, and more so for larger numbers than for smaller numbers. At a reasonable viewing distance, the bias would be small and mostly linear with the intended response location – but if anything its non-linearity would go opposite to the pattern predicted by the log scale (little increase over most of the scale, but a sudden increase for numbers on the right side of screen; see figure S2).

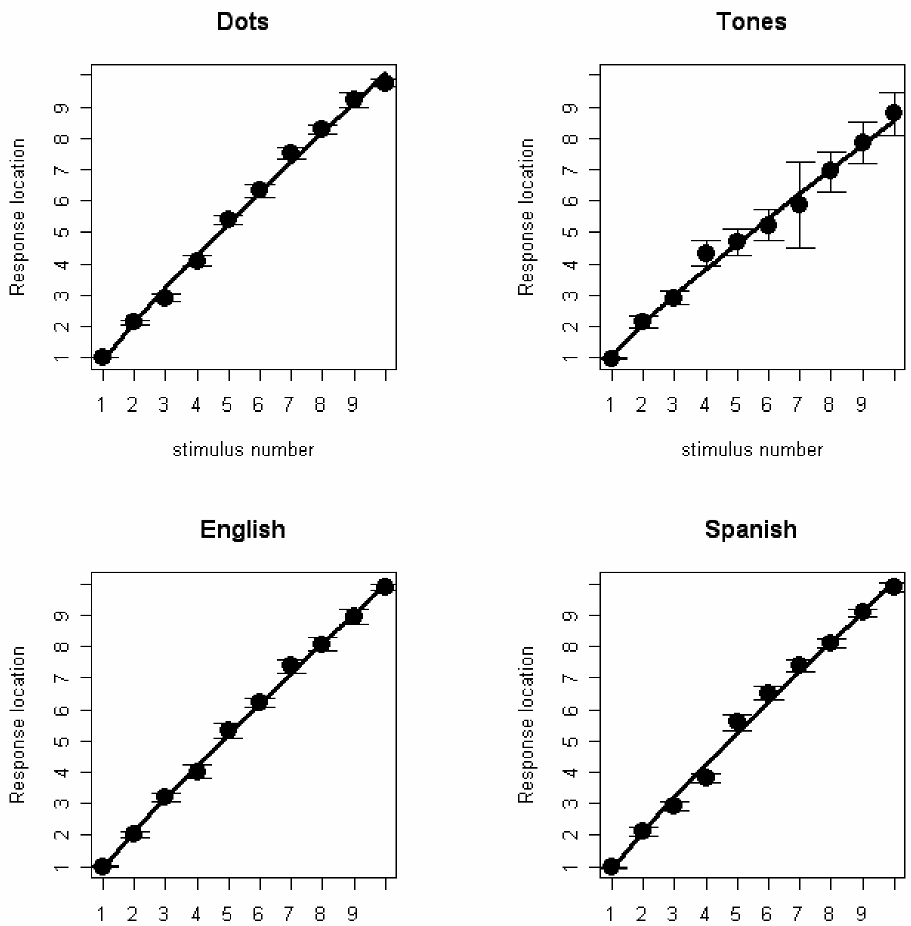
Figure S2 : diagram of putative parallax errors



Moreover, any systematic misperception of the intended response location could not have exceeded half the width of the pencil, or about 3 mm, which on a 25 cm line, translates into about 0.11 numerical units of the 1-10 response scale used in the plots of figure 2 (calculated as 0.3 cm / 25 cm * 9 response units). Yet even after linear regression of the participants' responses, the residuals (estimating of the amount of spatial displacement induced by the log scale) exceeded this putative measurement error by a large margin (overall residual = 0.44; dots: 0.61; tones: 0.32; Mundurucu words: 0.56; Portuguese words: 0.47). A residual of approximately half a numerical unit, as observed here, amounts to a shift of 14 mm on the screen, which we consider a very unlikely error given the present experimental conditions. Note also that the residuals for different target numbers are not distributed randomly, but rather are systematically arranged according to a log scale.

To directly address the issue of a differential bias in Mundurucu and American participants, we ran seven additional American adults using the same procedure as in the Amazon. The results appear below. They are virtually identical to those obtained when the American participants responded directly with the computer mouse, and do not show any hint of logarithmic responding comparable to what is seen in the Mundurucus' responses:

Figure S3 : Data from a control group of American adults who responded by pointing to the screen with a pencil, like the Mundurucu participants:



In summary, the patterns of response genuinely differ in the Mundurucu and the American participants, and this difference cannot be easily ascribed to artifacts of the experimental procedure.

2. Experimenter bias

Another possible methodological concern is that the experimenter perceived the same stimuli as the participant and could have introduced, voluntarily or involuntarily, a spatial shift when entering responses with the computer mouse. While we cannot completely exclude this possibility, it is unlikely for the following reasons:

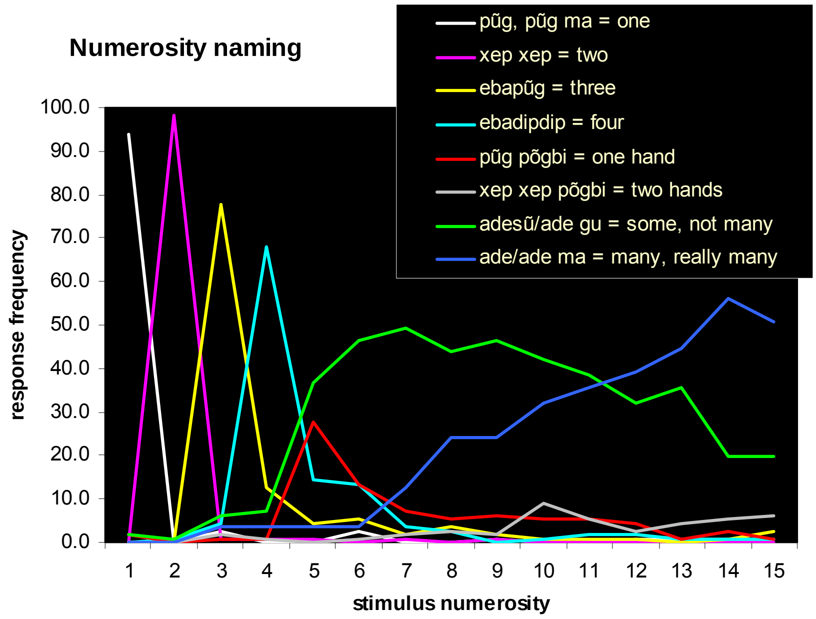
- The Mundurucu participants were tested by Pierre Pica, assisted by his wife Ana Arnor and by a native member of the Mundurucu culture who was bilingual in Portuguese and a good reader (and thus was able to name the Mundurucu words that appeared on screen). On the first trip, neither Pierre Pica whose main training is in linguistics, neither his spouse held strong hypotheses regarding the shape of the mapping from number to space. Moreover, given that P.P. is an educated Westerner, it seems likely that any unconscious influence of his cultural background would have shifted the results towards linear rather than logarithmic responding. Nevertheless the results of this first trip already exhibited a clear logarithmic effect. The hypothesis of experimenter bias also fails to explain the unexpected finding that the results changed with education level, but only for the Portuguese. It is hard to see how such a specific pattern of interaction with education level and with stimulus notation (a within-subject variable, thus varying across blocks within an experimental run) could have arisen from experimenter bias.

3. Logarithmic or bilinear responding?

One may question whether the observed response curves should be fitted by an alternative bilinear model with two separate linear regressions – a steep one for small numbers and a shallower one for large numbers. Such a bilinear function might mimic a logarithmic curve, and might be justified by the fact that the Mundurucu language only has relatively frequent names for numbers "one" to "five".

Contrary to this suggestion, however, our initial research indicates no discontinuity between small exact and large approximate numbers in the Mundurucu (*4*). The Mundurucu have names for numerosities 1-5, but even those names seem to behave like "round numbers" that refer to approximate numerosities. As shown in figure 1 of our previous paper, reproduced below as figure S4, the Mundurucu use the names 1-5 with decreasing frequency and increasing imprecision, for instance occasionally using the word for 5 to refer to 4 or to 6 objects. Thus, the Mundurucu lack a counting system and do not entertain exact concepts of number – their approximate system obeys Weber's law and seems to extend to small numerosities.

Figure S4. Numerosity naming in Mundurucu (from Pica et al. 2004, ref. 4)



Furthermore, although much research in Western children and adults supports a distinction between small and large numbers (*5, 6*), we know of no theoretical model that predicts that small and large numbers should be mapped onto space as two linear functions with distinct slopes (and with a smaller slope for larger numbers). By contrast, logarithmic modelling of the number-space task was proposed on a-priori grounds based on prior results from multi-dimensional scaling of the number dimension (*7, 8*), numerosity discrimination (*9*), numerosity naming (*10*), animal neural coding of number (*11*) and many other experimental results (*12, 13*).

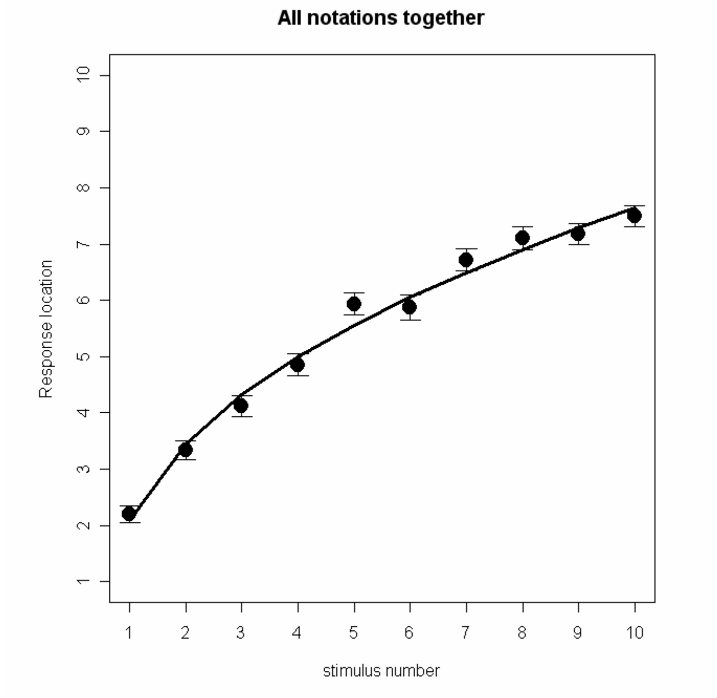
On the empirical side, the distinction between a logarithmic model and a piecewise linear model with two distinct linear ranges is hard to make, especially considering that we only have measurements in the range 1-10. Our attempts at data fitting resulted in extremely good fits for both models. Note, however, that the piecewise linear model uses four degrees of freedom (two slopes and two intercepts) while the logarithmic model uses only two. Furthermore, several further tests led to a clear rejection of specific versions of the bilinear model model. In all of these tests , we considered the natural hypothesis, derived from the developmental and linguistic literature, that the data should be separated into small numerosities smaller than 5, which have a relatively precise name in Mundurucu, and numerosities larger than 5. Since it is unclear whether 5 should be included in the former or the latter, we excluded it from this analysis.

- A first hypothesis that can be rejected is that participants respond linearly to numbers smaller than 5, and respond randomly to numbers larger than 5, which are less unfamiliar and for which they do not have precise names. This would predict a flat response curve for numbers larger than 5. However, in the range 6-10, the Mundurucu's mean spatial response increased monotonically with log numerosity. This effect, although small, reached significance in essentially all conditions of the experiment (averaged data: p=0.05; dots: p=0.028; tones: one-tailed p=0.045; Mundurucu words: one-tailed p=0.035; Portuguese words: one-tailed p=0.068).

- We then analyzed separately the data ranges 1-4 and 6-10, attempting to reject a linear model on both of these ranges. To reduce the variance, we averaged across all the stimulus notations (see figure below), and tested again the log versus linear models. On the range 6-10, the results were clear: the log regressor was significant even in a multiple regression with a linear component (p=0.016), and the log+linear model was significantly superior to the linear-only model (p=0.016). On the range 1-4, the results were similar though only approaching significance (log regressor, p=0.096; superiority of the log+linear model over the linear-only model, p=0.096). Note that there were only four degrees of freedom for this analysis, which therefore had little power.

Overall, these analyses provide consistent support for the logarithmic model, even when the data are analyzed with considerable scrutiny and over remarkably narrow ranges (and therefore with very few degrees of freedom). The figure below shows the remarkably good fit that the logarithmic model provides, when averaging across all the notations for numbers 1-10.

Figure S5. Average response location for all Mundurucu participants and all numbers 1-10 (averaged over the different presentation formats).

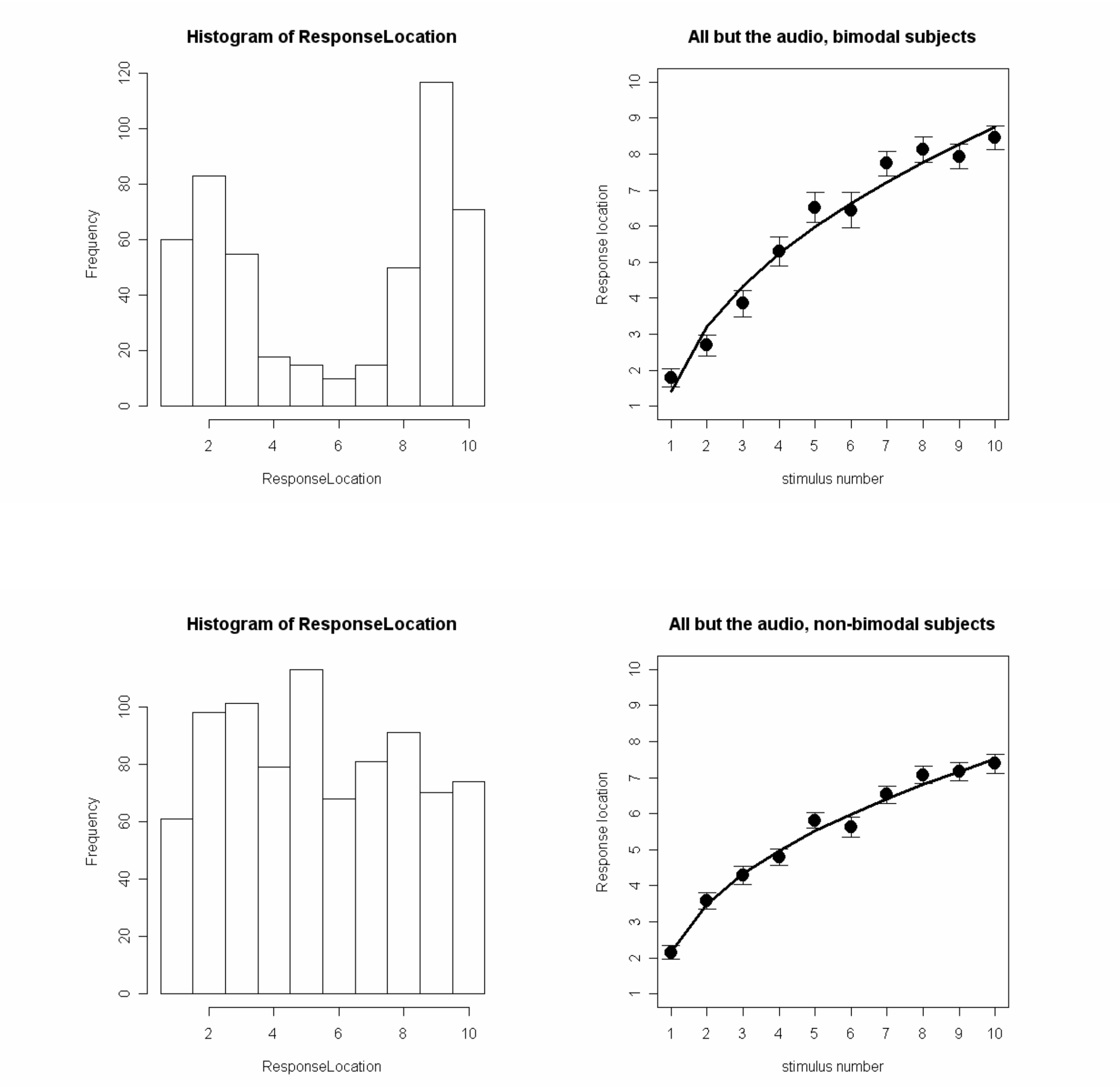


4. Unimodal or bimodal distributions of spatial responses

A final concern is that logarithmic responding could have arisen from participants' failure to use the full spatial response scale, but using only the endpoints in different proportions. We noticed that some Mundurucu participants tended to preferentially point towards the endpoints of the spatial scale, as if they were judging, for each target stimulus, whether it was more similar to the left endpoint (bearing the numerosity 1) or to the right endpoint (bearing the numerosity 10). In this case, the mean response location would correspond to the weighted mean of two relatively sharp response distributions and would thus be a rather misleading index of the actual spatial location of the participants' responses. Logarithmic responding would arise, not because the participants think of numbers as extending in space in a compressive manner, but because they rate the similarity of two numbers based on their ratio rather than based on their linear distance.

To clarify this point, for each participant's experimental run, we computed a histogram of the spatial location of the responses to numbers 1-10. We then classified the runs as 'unimodal' or "bimodal' depending on whether the percentage of responses falling in the middle of the response segment (corresponding to the linear range of numbers [3.5, 7.5], and thus to 4/9 of the linear scale 1-10) was above or below 20%. A majority of experimental runs (22 out of 35) were classified as unimodal, indicating that most participants followed the instruction of using the entire extent of the response scale. However 13 runs were classified as bimodal. As shown below, however, logarithmic responding held whether or not the participants' responses were distributed unimodally on bimodally on the response continuum (these plots are averages across the dots, Mundurucu and Portuguese stimuli 1-10, excluding the tones stimuli for which the American participants also responded logarithmically). Regressions revealed a compressive, logarithmic effect in all cases.

Figure S6. Distribution (left) and mean (right) of the response location for numbers 1-10 (all formats except the tones), plotted separately for experimental runs showing evidence of bimodal (top row) or unimodal (bottom row) responding.



III. Mean number-space mappings for different subgroups

Figure S7. Mundurucu adults only

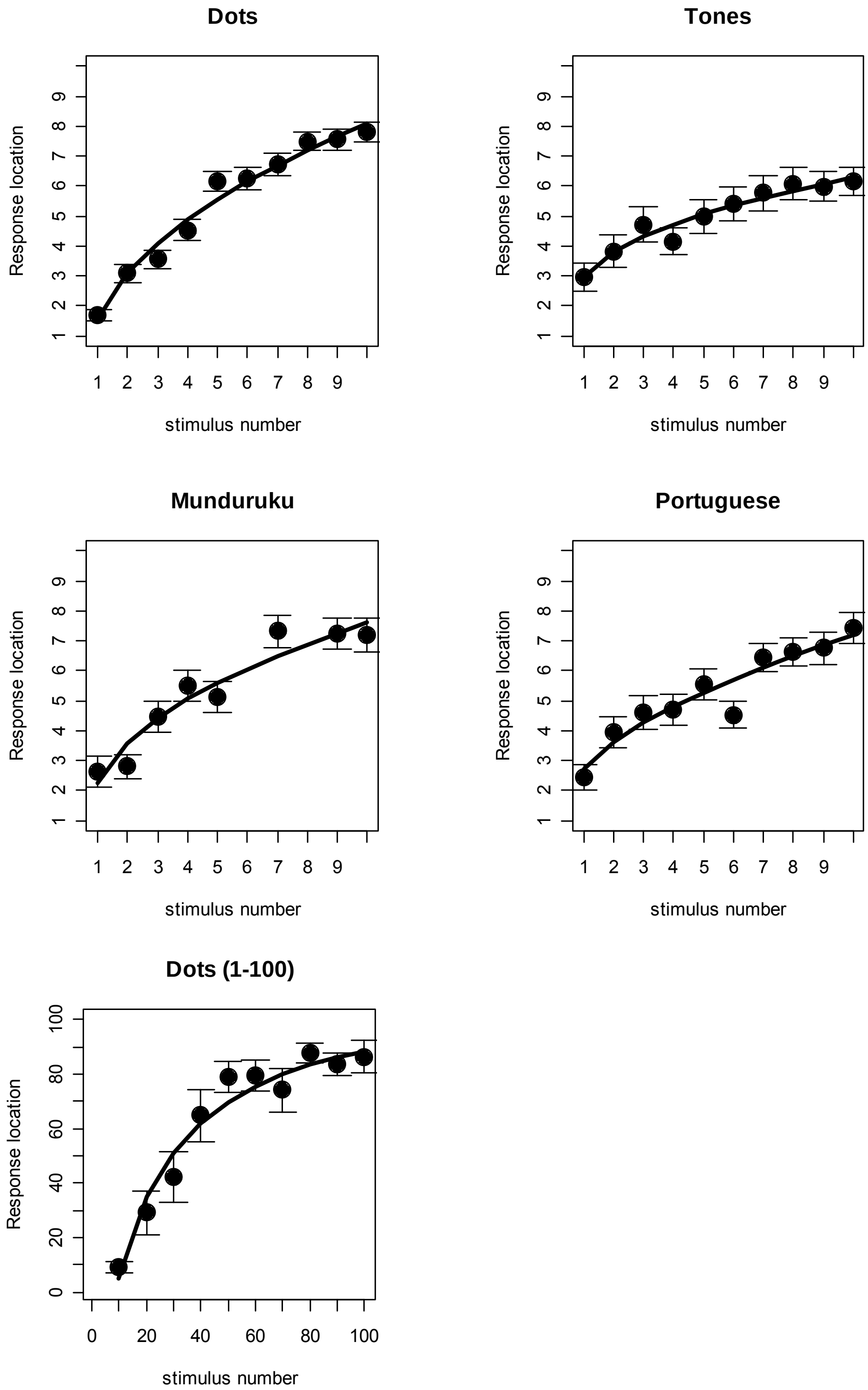


Figure S8. Mundurucu uneducated adults only

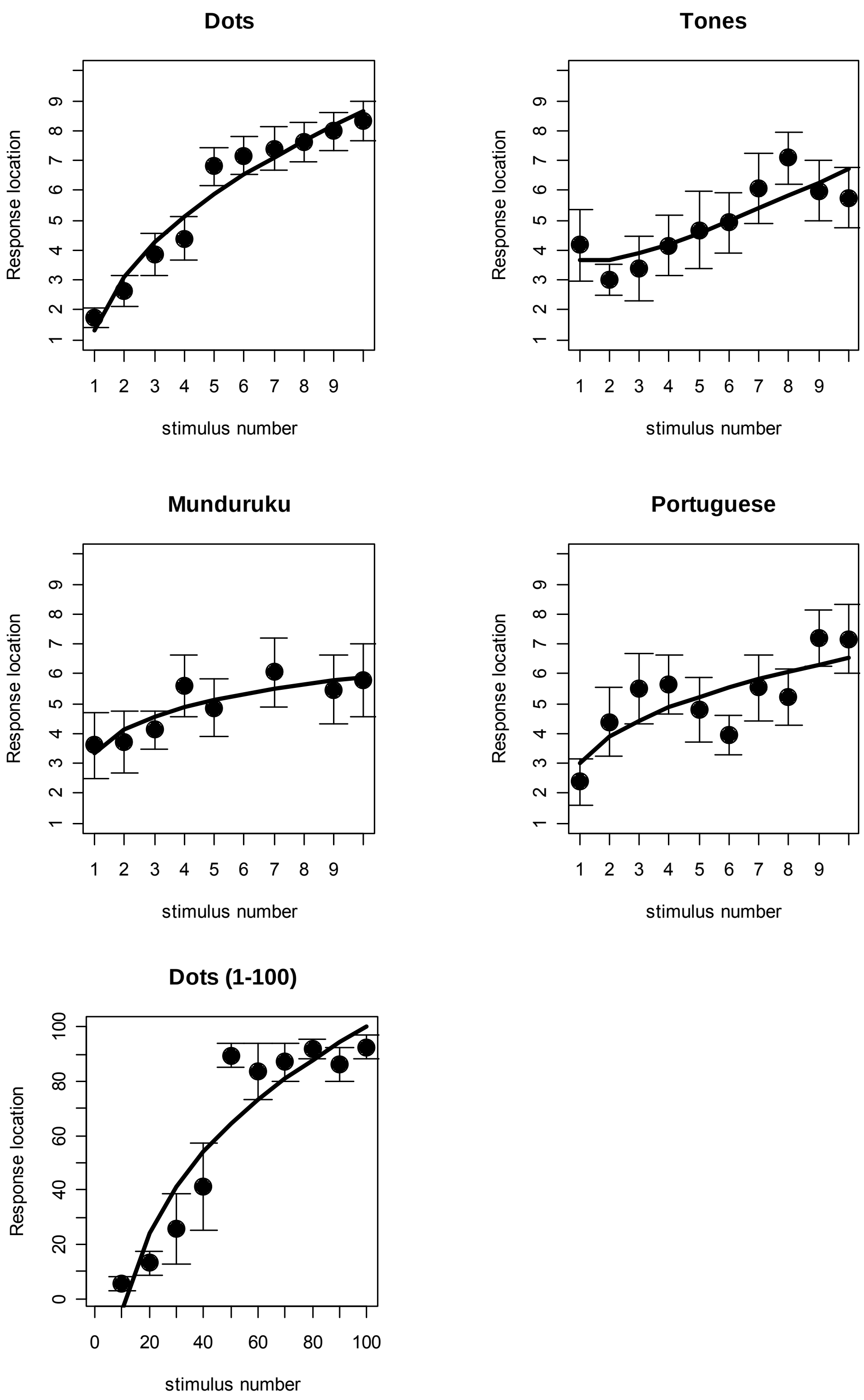


Figure S9. Mundurucu subjects with little education (years 1-2)

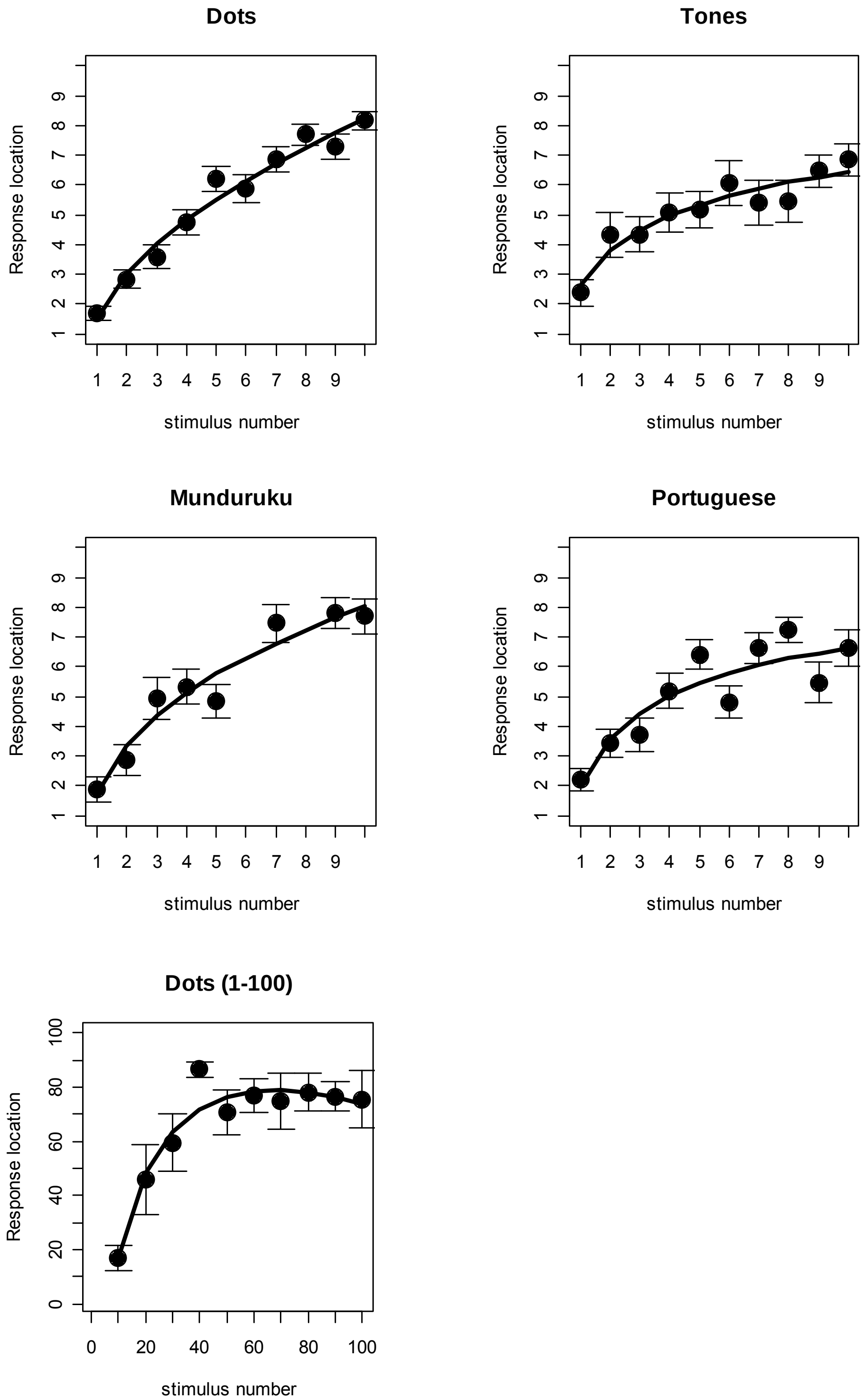
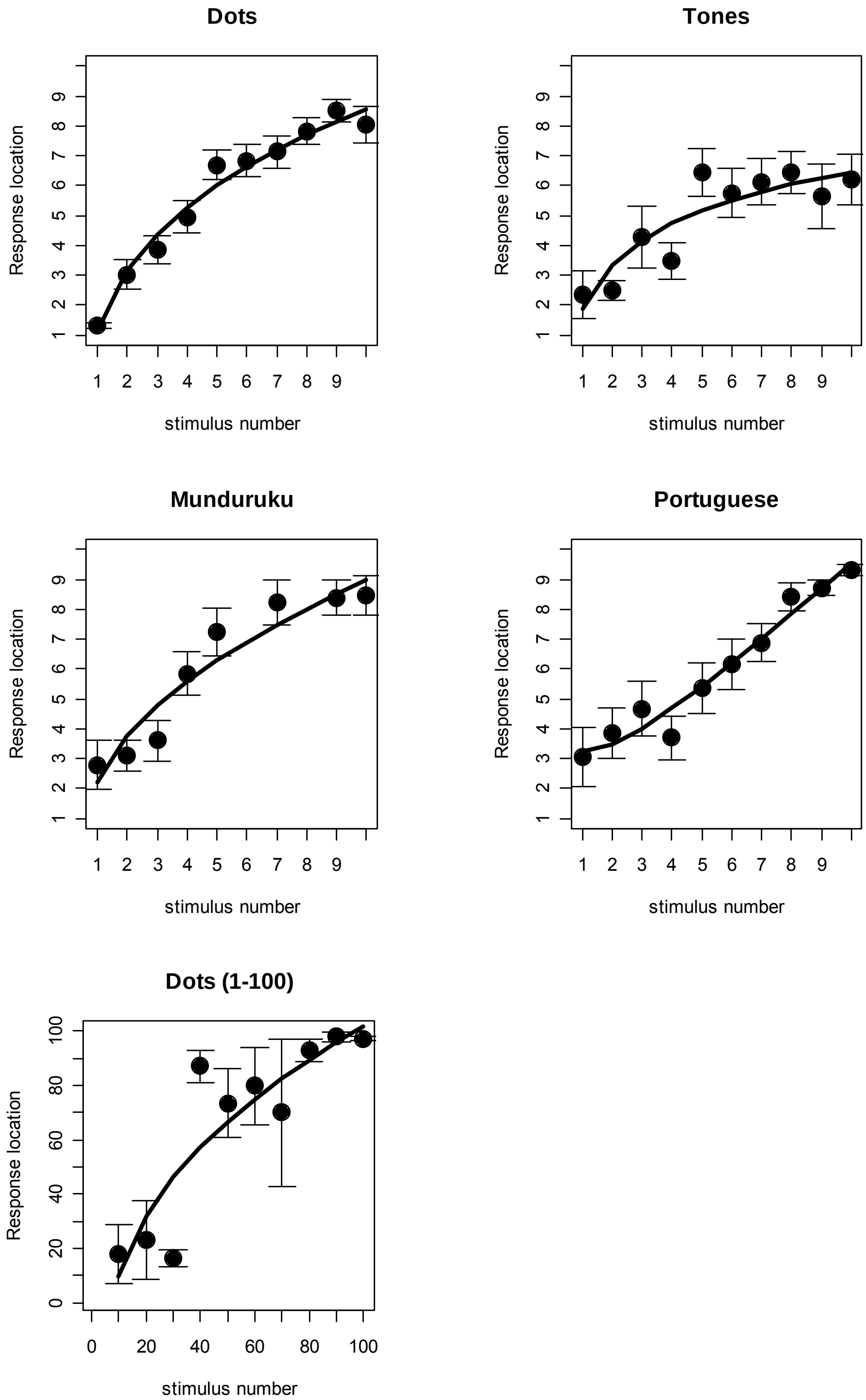


Figure S10. Mundurucu subjects with more education (years 3 and above)



IV. References for supplementary material

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